Two Influences of Trafalgar on Navy Tactics 1905-1944

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Abstract

Nelson's victory off Cape Trafalgar in 1805 had two major influences on subsequent Navy tactics. Raking cannon fire at close range developed into Crossing the T with rifled guns at long range at Tsushima in 1905. Dividing the battle line with superior gunnery stimulated quantitative analysis of battle attrition and led to Lanchester's equations in 1916. Although raking fire had occurred earlier against barbary corsairs and de Ruyter had cut and doubled the battle line, it was Nelson's dramatic unification of these tactics that captured the attention of subsequent naval strategists. His cutting off the van generated great tactical power, only recently appreciated.

Keywords: Raking Fire, Crossing the T, gunnery, tactics, Lanchester's equations

Acknowledgements.

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This study had available many excellent sources. Adkin's Trafalgar Companion is an encyclopedia of data besides being the standard in-depth analysis of the battle, whence much here is generously borrowed. Lanchester's 1916 Aircraft in Warfare was the basis of the numerical analysis. It covers naval tactics as wells as the future of aviation. Mahan, Morison, Hughes and Hornfischer are well-known classics and were used for specific actions. The Osprey publications were helpful. Wikipedia and Google also have been invaluable for data, illustrations and explanations of calculations.

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Raking Fire

In the 18th and 19th centuries, the usual type of naval gun battle was broadside-to-broadside from two parallel battle lines. Both fleets hurled the maximum amount of metal at each other. Symmetric attack was the doctrine. The doctrine of asymmetric attack developed with raking fire. During a rake, one ship shot a broadside down the length of another ship. A rake fired through an unprotected spot, at a short distance with high accuracy, hitting everything in the ship.

Raking Fire before Trafalgar

Raking fire was used before Trafalgar. Figure 1 shows firing 'in enfilade' down an infantry line.

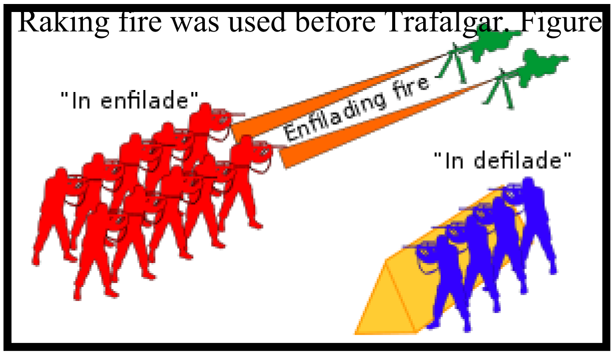

Figure 1. Enfilade fire. Raking adapted the force multiplier of this concept. Wikipedia.

Enfilade fire was first documented as early as the Battle of Taginae in June/July 552 AD/CE when the forces of the Byzantine Empire under Narses defeated King Totila of the Ostrogoths in Italy. Byzantine archers amassed on the flanks of their army to incline the front towards the center so that their battle line became crescent shaped. The charging Ostrogoth cavalry was caught in the enfilading fire from both sides, with high casualties.

In naval battles, the bow and especially the stern of a ship is vulnerable to a broadside, noted apparently early on. There have been allusions to barbary corsairs using raking fire to disable merchant ships possibly from a Xebec.[1] The author, however, searched for early depictions of this without success. The encounter documented in Bonaventure Peeters' **1645** painting in Figure 2 shows the wind blowing flags from left to right impeding the corsair but

Figure 2. **Dutch Naval Action between the Flagship Zevolle and Barbary Corsairs.** Peeters, Bonaventure (Flemish) (1614-1652). 1645. Seascape Gallery. Channel Islands Maritime Museum, Oxnard, CA. FZ rakes stern of BC. Wind blowing left to right, this way → . (Low resolution.)

filling the sails of the Dutch ship, which rakes the corsair's stern, rather than vice-versa. This is the earliest depiction of a ship raking another ship, in the seventeenth century in a battle of Dutch against Corsairs. Thus, raking fire could have been well-known 160 years before Trafalgar.

In **1780** after several defeats, it was the French, ironically, who changed their tactics to raking fire against the English in defense. The French took the lee gage, downwind position to

[1] The Xebec was a large, fast, open craft with multiple cannon, powered by lateen sails and oars, and designed to close with unarmed merchants in the Mediterranean. There is a scale model in the CIMM Seascape Gallery. Channel Islands Harbor, CA.

await an English attack. As the English came in a column abreast, the French raked the bows of the English ships with broadsides.[2]

Raking Fire at Trafalgar

Nelson was aware of this French defensive tactic at Trafalgar. But he also knew that the French-Spanish fleet had little time to practice gunnery, bottled up in Cadiz Harbor. He anticipated only one or two salvos as he approached head on, when they were in a position to 'cross his T.' He was right.[3]

Villeneuve was aware of the details of Nelson's plan of attack in advance.[4] In a memorandum to his staff in anticipation before the battle, Villeneuve wrote:

> "The British Fleet will not be formed in a line of battle parallel to the combined fleet according to the usage of former days. Nelson, assuming him to be, as represented, really in command, will seek to break our line, envelop our rear, and overpower with groups of his ships as many as he can isolate and cut off."[5]

Knowing this, it is surprising that Villeneuve would disorganize his battle line for hours turning back to Cadiz right before the battle. This would reduce the power of Villeneuve's defensive tactic. The reason given by historians was concern that Nelson might cut him off from Cadiz. The wearing back through 180 degrees in a very light wind with heavy swell with the British at six miles was a decision bemoaned by one Spanish Captain: "The fleet is doomed…He has compromised us all."[6]

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[2] Lanchester FW. Aircraft in Warfare: 62.
[3] Adkin M. The Trafalgar Companion: 275.
[4] Lanchester FW. Op.cit.: 63.
[5] Ibid: 63.

This maneuver prevented a tight battle line with optimal concentration of cannon. When Nelson ran the gauntlet, fewer enemy ships could hit him with bow-on, defensive raking fire.[7] In the actual battle, eight ships fired 850 shots from their broadsides at Nelson's advancing column. Of these, 15 percent or 130 hit the Victory. They destroyed the wheel, the mizzen top mast, the foresail, the studding sail and inflicted 9 casualties. Half of the 350 guns firing were out of the 500-yard effective range of the cannon. Only four ships of the Combined Fleet fired effective shots, about 37% were on target.

Figure 3 shows stern raking fire at Trafalgar. Nelson's flagship, the Victory, passes just astern of Villeneuve's flagship, the Bucentaure. Due to the low wind, the Victory moves at 1.5 knots, only 15 feet from the stern of the Bucentaure. It takes 73 seconds for the 186 feet of the Victory to pass the 50 feet of the stern. Victory has enough time to carefully aim and fire each of the 51 cannon of her left broadside directly through Bucentaure's stern, quarterdeck, rear

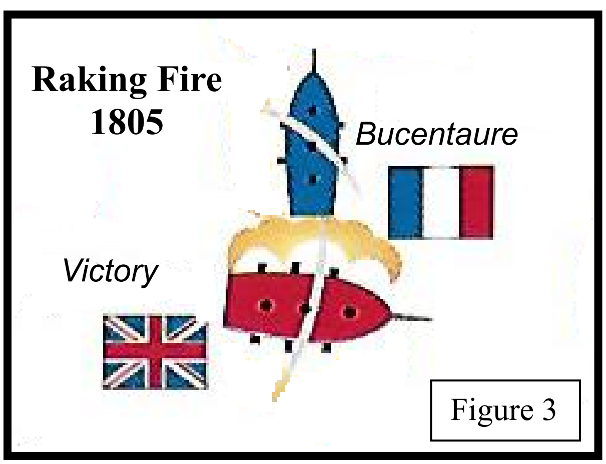

Figure 3

windows and rudder, down the long axis of the ship. The devastating maneuver kills 197 crew, wounds 85, disables cannons, dismasts the ship, destroys her rudder and decapitates command and control, all in one masterly stroke.

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[6] Adkins M. Op.cit.: 477.
[7] Ibid.: 475-478

Crossing the T

French defensive raking fire on an approaching column suggests the beginning of Crossing the T. The full power of raking was shown by Victory on Bucentaure. Certainly, some armchair captains must have wondered '*What if* the French had a little more gunnery practice?' '*What if* the Combined Fleet had just trained to Nelson's standard of gunnery rate of fire with their own accuracy of a 37% hit rate?' '*What if* Villeneuve hadn't lost his nerve, turned and diluted his force at the last minute?' … the Victory could have sustained a lot more broadsides… A game-changer?

Shooting down the long axis accurately with low risk.

The same principles operated in Victory's rake on Bucentaure and the French rake on Nelson's approaching column. Maximum metal from a broadside down the length of an opposing ship. The direction of firing was through an unprotected area, the bow or especially the stern. At this point, the opposing ship had minimal or no cannon to fire back at the attacking ship. Maximum destruction, unopposed.

Over the century from 1805 to 1905, Britain maintained peace on the high seas. Little naval warfare or advance in technology. The only significant exception was the Battle of Lissa off the coast of Dalmatia in 1866 during the Austro-Prussian War. A smaller Austrian fleet crossed the T of a larger Italian force and defeated it. But ramming destroyed most enemy ships, a tactic not generally useful afterwards. Unfortunately, the success of ramming distracted a generation of naval strategists from the importance of gunnery.[8] With the appearance of modern

[8] Wikipedia entry

rifled guns with a range of 10 miles and fire control at the end of the 19th Century, 'Crossing the T' developed into an overwhelming stratagem. Enter the age of steel battleships.

How Crossing the T works.

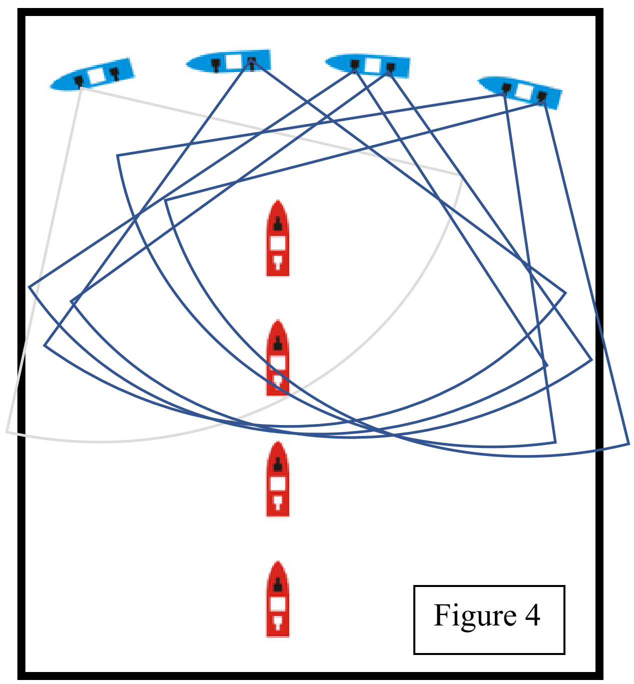

Figure 4

Figure 4 shows a diagram of Crossing the T. The distance between the two forces usually is about 10 miles, but here the distance has been collapsed in the diagram to clarify the interactions. The Blue Force has eight (8) guns and the Red Force has eight (8) guns. The Blue Battle Line is crossing the Red Column's T. All eight (8) of Blue's guns can fire a broadside at Red. The first two Red ships are within a 10-mile range of Blue's rifled 16-inch guns. Note that at 10 miles all four Blue ships would see about the same end on view of the Red column, rather than the partial side view in the simplified diagram. The Red Column therefore is really an extended target line down range. Thus, the Blue guns have a simpler fire control problem. They need to determine precisely the azimuth setting for their sights. But they need fewer ranging shots to hit a target, since most will fall on a ship in the extended red column. Thus, Crossing the T concentrates maximum firepower onto the enemy accurately, with smaller range error.

For the Red Column, on the other hand, the situation is worse. Figure 5 shows that it can bring only two (2) guns to bear on the Blue Battle Line. This is because the stern guns cannot pivot far enough forward to aim at the Blue ships. The Red ships in the rear of the column are either out of range or afraid to fire lest they hit their own ships ahead. Furthermore, the Red Column is firing at a distant, horizontal, thin line. It is a non-extended target, the cross bar of the T. Red will need to fire more ranging shots to hit. It is at a disadvantage.

In summary, the Blue battle line as it crosses the T has a four-to-one (4:1) advantage in firepower, the effectivenes ranging error.

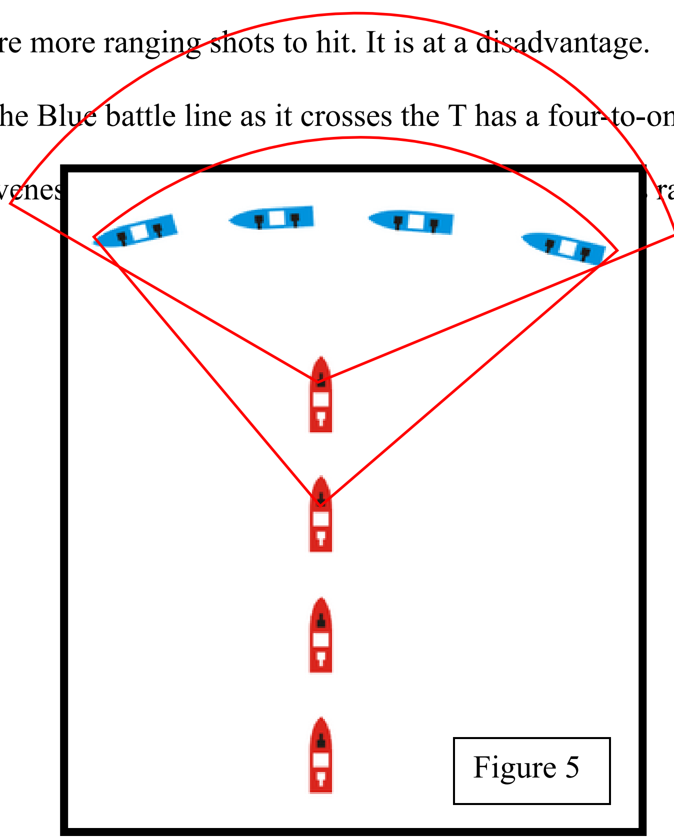

Figure 5

Divide and Conquer

A larger force has an advantage over a smaller force, all else being equal. On the morning of 21 October, Nelson had 27 ships-of-the-line with 2150 guns. Villeneuve had 33 ships-of-the-line with 2624 guns. The smaller ships performed other duties. Villeneuve had a numerical advantage. Nelson soon reduced Villeneuve's ships-of-the-line and applied superior gunnery.

Breaking the Line before Trafalgar

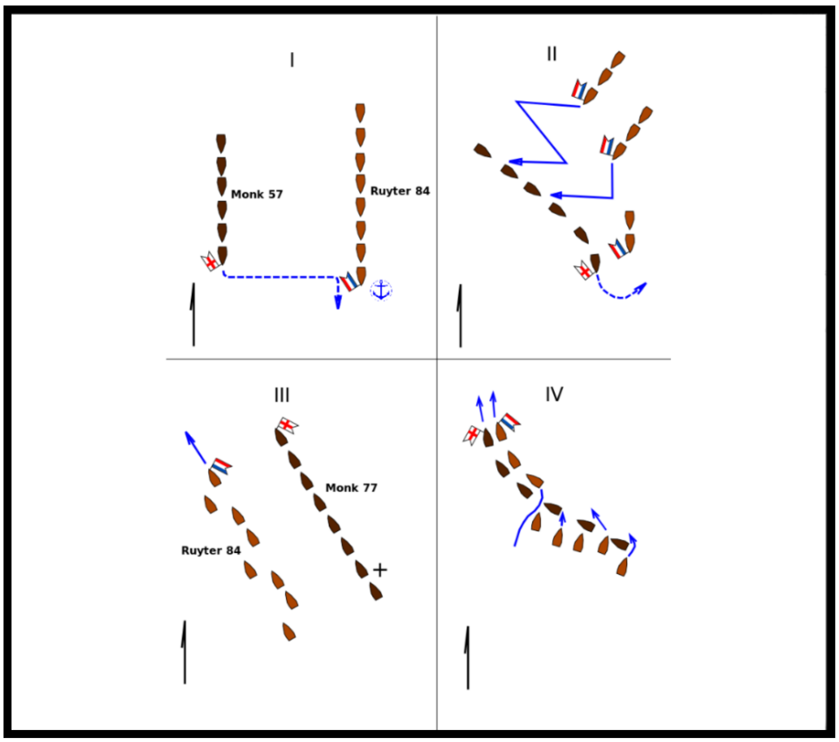

Figure 6. The Four Days' Battle of the Se War. IV. The fourth day. De Ruyter cuts of Monck. Military.wikia.org/wiki/Four_

The Second Anglo-Dutch War pitched two great Admirals at each other - Monck for England and de Ruyter for Holland.[9] On 4 June 1666, de Ruyter cut the battle line of Monck during the Four Day's Battle (Figure 6 IV). This appears to be the first instance of cutting the battle line of an opponent.

Nelson cuts off the Van

[9] Mahan AT. The Influence of Sea Power upon History 1660-1783. (originally Little, Brown, Co., Boston, 1890). Reprinted Dover, New York. 1987:117-125.

The two British columns bisected the Combined Fleet (Figure 7). This isolated the ships of the

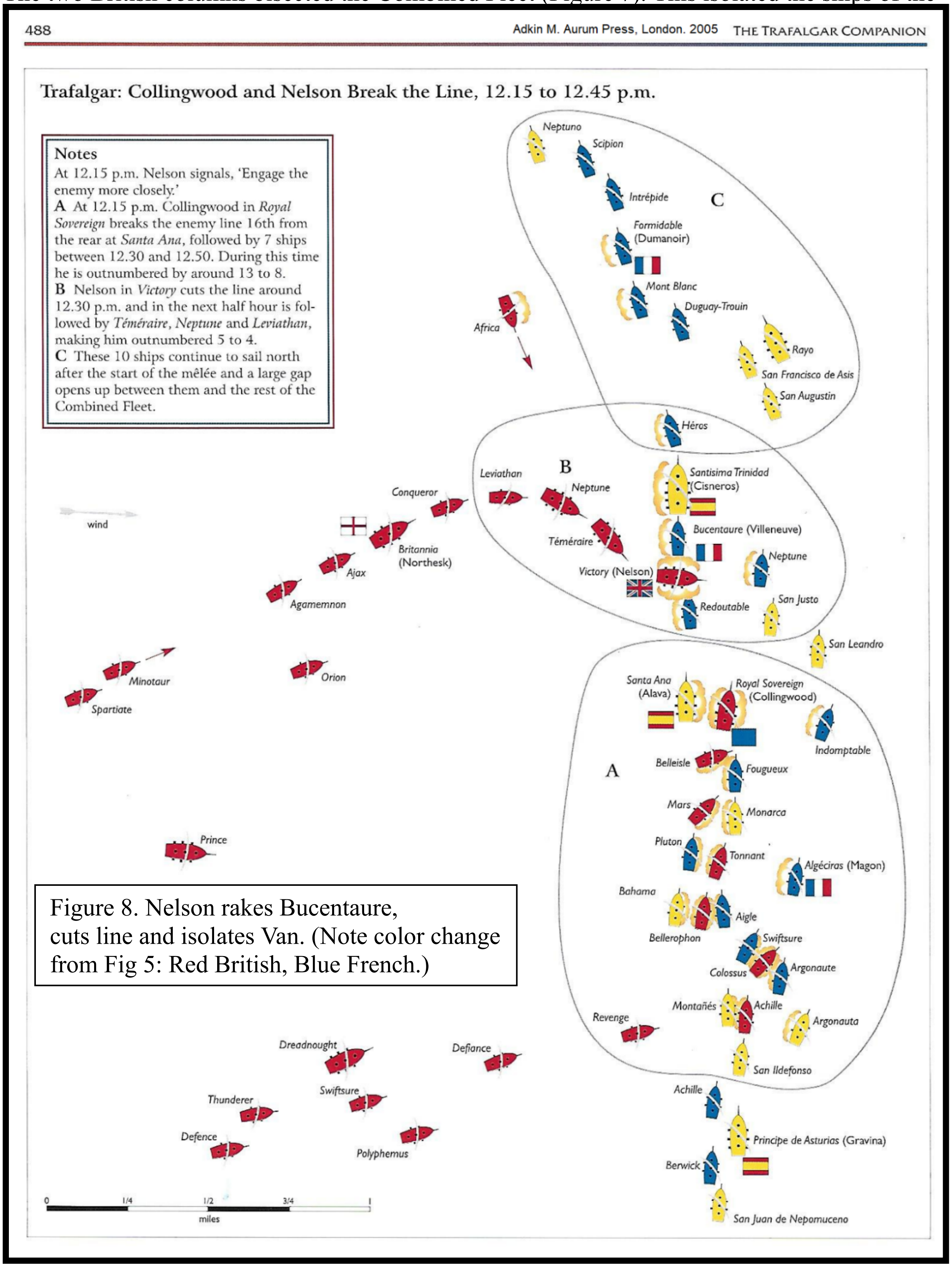

Figure 8. Nelson rakes Bucentaure, cuts line and isolates Van. (Note color change from Fig 5: Red British, Blue French.)

Nelson thus added to his numerical advantage. Table 1 shows the details. Nelson had 27 ships-of-the-line with 2225 total guns. After the van was isolated, the combined fleet shrank from 33 ships-of-the-line with 2759 total guns to 23 ships with 1945 guns. Immediately Victory raked Bucentaure. In the next 90 minutes, Bucentaure lost 84 guns and Redoutable and Fougueux followed with a loss of 156. By then, the British had destroyed three ships-of-the-line, the flagship with commander and 240 guns, reducing the enemy to 1687 guns.

BRITISH FLEET	Ra	*"Guns"*	Can	Car	Total	COMBINED FLEET	Ra	*"Guns"*	Can	Ob	How	Total
Victory	1	*102*	102	3	105	Neptuno	3	*72*	72		18	90
Téméraire	2	*98*	98		98	Scipion	3	*74*	74	4		78
Neptune	2	*98*	98		98	Intrepide	3	*74*	74	4		78
Leviathan	3	*74*	74		74	Formidable	3	*80*	80	6		86
Conqueror	3	*74*	76	6	82	Mont Blanc	3	*74*	74	4		78
Britannia	1	*100*	100		100	Duguay-Trouin	3	*74*	74	4		78
Africa	3	*64*	64		64	Rayo	1	*100*	100			100
Ajax	3	*74*	64	8	72	San Fran de Asis	3	*74*	74			74
Agamemnon	3	*64*	64		64	San Augustin	3	*74*	74			74
Orion	3	*74*	74		74	Héros	3	*74*	74	4		78
Minotaur	3	*74*	74		74	**Sntssa Trinidad**	1	*136*	126		10	136
Spartiate	3	*74*	62	20	82	**Bucentaure**	3	*80*	80	4		84
Roy Sovereign	1	*100*	96	8	104	**Redoutable**	3	*74*	74	4		78
Belleisle	3	*74*	64	18	82	**San Justo**	3	*74*	74			74
Mars	3	*74*	74		74	**Neptune**	3	*80*	80	6		86
Tonnant	3	*80*	66	18	84	**San Leandro**	3	*64*	64			64
Bellerophon	3	*74*	74	8	82	**Indomptable**	3	*80*	80	6		86
Colossus	3	*74*	64	18	82	**Santa Ana**	1	*112*	112		6	118
Achille	3	*74*	66	18	84	**Fougueux**	3	*74*	74	4		78
Revenge	3	*74*	74	8	82	**Monarca**	3	*74*	74			74
Defiance	3	*74*	74		74	**Pluton**	3	*74*	74	4		78
Swiftsure	3	*74*	62	18	80	**Algéciras**	3	*74*	74	4		78
Dreadnought	2	*98*			98	**Bahama**	3	*74*	74		10	84
Polyphemus	3	*64*			64	**Aigle**	3	*74*	74	4		78
Thunderer	3	*74*			74	**Montañés**	3	*74*	74			74
Defence	3	*74*			74	**Swiftsure**	3	*74*	74	4		78
Prince	2	*98*	100		100	**Argonaute**	3	*74*	74	3		77
						Argonauta	3	*80*	72		8	80
						San Ildefonso	3	*74*	74	12		86
						Achille	3	*74*	74	4		78
						Prin de Asturias	1	*112*	118		6	124
						Berwick	3	*74*	74	4		78
						SJ Nepomuceno	3	*74*	74			74
[redacted]												
						23		*1854*		***63***	*40*	***1945***

Table 1: Ships and Guns at Trafalgar. "*Guns"* (*stated armament*). Can (cannon). Car (carronade). How (Howitzer). Total (actual armament). *Ra* (rating, Howitzers not included in rating). First ten (10) ships of Combined Fleet in Van cut off at start. Last line shows Combined Fleet after van cut off. Extracted from Adkin M. Tr Co: 307-393.

The van meantime began to wear at 2:00 pm and returned after 3:00 pm. But only three of the ten ships then engaged the enemy: Formidable, Intrépide and San Augustin with their 238 guns. The British had superior numbers for the first hours of the Battle.

Gunnery and Guns

British firepower was superior to that of the French and Spanish. Why revisit this after all the prior literature on Trafalgar? The reason is Nelson's disproportionate gain in firepower when he isolated the van, 30% of Villeneuve's fleet. Lanchester analysis below will show that Nelson instantly doubled his in firepower by this maneuver. The following background is helpful for that analysis.

Gunnery

Accuracy of fire was apparently not grossly different between the opposing fleets. The British fired at the ship's hull, while the Combined fleet fired at the masts and rigging. The Combined fleet fired 850 shots at the Victory as it closed on the battle line. Of these, 350 shots were fired in range, of which an estimated 130 hit the ship (37%). Aim was degraded by heavy swells hitting beam on, roll causing variable range error. So, these results may underestimate the Combined fleet's average ability.[10] Accuracy was especially important in a standard encounter of battle line to battle line at a distance.

Rate of fire was a different story. It was of primary importance in the mêlée, the subsequent, decisive phase of battle, when ships closed and slugged it out next to each other. The British kept the Combined fleet bottled up in Cadiz most of the time. The French and Spanish had to use the little sea time available to practice seamanship. Seamanship was ingrained in the British, an island nation. They practiced it daily to keep the enemy fleet at bay.

Nelson ensured regular gunnery exercises. The result was that a British gun crew could fire three shots in five minutes. A less experienced Combined fleet gun crew could fire a 32

[10] Adkin M. The Trafalgar Companion: 486

pounder every eight minutes, an 18 pounder every five minutes and a 9 pounder every 4 minutes.[11] One can compare three gun crews manning 32-, 18-, and 9- pounders in the Combined fleet firing against their three counterparts in the British fleet. In 40 minutes ('), the opposing gun crews could fire:

British: 3x(3shots/5min) x 40min = (9sh/5')x40' = 72 shots,

Combined: (1sh/8m)x40m + (1sh/5')x40' + (1sh/4')x40' = 5+8+10 = 23 shots.[12]

A Ratio of = British/Combined = 72/23 = 3.13

This overwhelming advantage in rate of fire would have won Trafalgar by itself. But there was the added advantage from cutting off van.

Guns

There are several ways to compare the firepower of the guns of the two fleets: number of ships of the same rating, total number of guns, pounds-of-metal-per-ship/broadside or one of the preceding with an adjustment for type of gun effect, such as anti-personnel. Table 1 shows that the total number of guns was not always implicit in the rating. A third-rate had from 64 guns (Africa) to 84 guns (Achille). Thus, "rating" can be misleading. Total number of guns mixes together weapons of different poundage. In the mêlée, as distinct from the battle line, all guns are in range and effective. There also are different types of guns. The carronades are very effective anti-personnel weapons, especially devastating at short range. The carronades are included in the gun totals for the British and the obusiers and howitzers for the French and Spanish.

Pounds-of- metal-per-ship can reduce this complex array of armament to one simple, objective number. This is a primary number. It can be extracted directly from the thorough

[11] Adkin M. The Trafalgar Companion: 268.

[12] From the same reference, another estimate for the heavier guns was four (4) minutes between broadsides. At this rate, it would be 30shots/40min – a ratio of 72/30=2.40, in the same range. Ibid.: 485.

analysis and data in Adkin.[13] Table 2 shows the number of each size and type gun with the total pounds-of-metal for each. The total pounds-of-metal at noon before battle were:

British fleet: 49,578 lbs. Combined fleet: 66,164 lbs. Ratio: 3:4 favors Combined fleet.

[13] Adkins M. The Trafalgar Companion: 307-393.

BRITISH Ship	Ca	Ca	Ca	Ca	Ca	Ca	Cr	Cr	Cr	Cr	Carronac	Guns +	COMBINED FLEE	Ca	Ca	Ca	Ca	Ca	Ca	Ca	Ca	Ob	Ob	Ho	Ho	Guns +	Not van
	32	*24*	*18*	*12*	*9*	*6*	*68*	*32*	*24*	*18*		Carron		*36*	*32*	*24*	*18*	*12*	*8*	*6*	*4*	*36*	*32*	*32*	*24*	Ob+How	
Victory	30	28		44			2			1	3	105	Neptuno		30		32	10							18	90	
Téméraire	28		60	10								98	Scipion	28		30			16			4				78	
Neptune	28		60	10								98	Intrepide	28		30			16			4				78	
Leviathan	28		28		18							74	Formidable	30		32			18			6				86	
Conqueror	30		30		16					6	6	82	Mont Blanc	28		30			16			4				78	
Britannia	28	28		28		16						100	Duguay-Trouin	28		30			16			4				78	
Africa		26	26		12							64	Rayo	30		32	30		8							100	
Ajax		56			8			8			8	72	San Fran de Asis		28		30		16							74	
Agamemnon		26	26	12								64	San Augustin		28		30		16							74	
Orion	28		28		18							74	Héros	28		30	16					4				78	
Minotaur	28		28		18							74	**Sntssa Trinidad**	34		34		34	18		6				10	136	136
Spartiate	28		34					20			20	82	**Bucentaure**	30		32		6		12		4				84	84
Roy Sovereign	28	28		40						8	8	104	**Redoutable**	28		30	16					4				78	78
Belleisle	30	30			4			14	4		18	82	**San Justo**		28		30		16							74	74
Mars	28	30			16							74	**Neptune**	30		32		18				6				86	86
Tonnant	32		34					18			18	84	**San Leandro**			28	30		6							64	64
Bellerophon	28		28		18			2		6	8	82	**Indomptable**	30		32		18				6				86	86
Colossus	28	36						10	2	6	18	82	**Santa Ana**	30		32		32	18						6	118	118
Achille	30	36						10	2	6	18	84	**Fougueux**	28		30			16			4				78	78
Revenge	30		30		14			2		6	8	82	**Monarca**			28	30		16							74	74
Defiance	28		28		18							74	**Pluton**	28		30			16			4				78	78
Swiftsure	28		34					12		6	18	80	**Algéciras**	28		30			16			4				78	78
Dreadnought	28		60	10								98	**Bahama**		28		30		16					10		84	84
Polyphemus		26	26	12								64	**Aigle**	28		30			16			4				78	78
Thunderer	28		28		18							74	**Montañés**		28		30		16							74	74
Defence	28		28		18							74	**Swiftsure**	28		30			16			4				78	78
Prince	28		30	30		12						100	**Argonaute**	28		30			16			3				77	77
													Argonauta		30		32	10						8		80	80
													San Ildefonso		28	46							12			86	86
													Achille	28		30			16			4				78	78
													Prin de Asturias	36		32		32	18						6	124	124
													Berwick	28		30			16			4				78	78
													SJ Nepomuceno		28		30		16							74	74
27											**151**	**2225**	33									77			40	2759	**1945**
																											1725
No of guns	lb	lb..									lb metal	factored	No of guns	lb	lb..											lb metal	factored
658	32										21,056	21,056	642	36												23,112	23,112
350		24									8,400	8,400	256		32											8,192	8,192
646			18								11,628	11,628	734			24										17,616	17,616
196				12							2,352	2,352	412				18									7,416	7,416
196					9						1,764	1,764	160					12								1,920	1,920
28						6					168	168	390						8							3,120	3,120
2							68				136	544	12							6						72	72
96								32			3,072	12288	6								4					24	24
8									24		192	768	77									36				2,772	5544
45										18	810	3240	12										32			384	768
											49,578	62,208	18											32		576	864
Increased fleet firepower with carronades' factor x4)												1.2548	40												24	960	1440
CarronadeMetal/TotalGunMetal											8.5%															66,164	70,088
CarronadeFirepower/TotalGunFirepower												27.1%															1.05931
Before the van is isolated											49,578	62,208	**Before** the van is isolated													66,164	70,088
												1.0000															**1.1267**
											βB^2=	**3.13**														**αF^2=**	**1.27**
											Adv	2.47															
													Van of 10 ships														
													200	36												7,200	7,200
													86		32											2,752	2,752
													214			24										5,136	5,136
													138				18									2,484	2,484
													10					12								120	120
													122						8							976	976
													26									36				936	1872
																										19,604	20,540
																											29%

Table 2. Numbers of each gun type for each ship at Trafalgar, above yellow. Below, total pounds metal of gun type, 'factored' column accounts for effect of carronades, obusiers and howitzers. Right lower part shows Van.

Carronades

In addition to cannon, the British had carronades. These were large bore, short barrel and lighter weight artillery pieces. They were mobile and could be moved around on the top decks to greatest effectiveness. They fired a heavy shot up to 68 lbs. with low velocity over short range.[14] Some authors state that the French obusiers functioned somewhat like carronades, but that Napoleon considered them inferior and badgered his war minister to supply carronades to his fleet:[15]

> "It was with carronades that the English set L'Orient on fire [at the Nile in 1798 and not necessarily true] and in them they have an immense advantage over us…But, for God's sake, ship me some carronades…"[16]

Others remark that the French "obusier de vaisseau" was a type of howitzer and confirm that it never provided the power of the British carronade.[17]

The carronade was the most effective anti-personnel weapon in the mêlée. Table 1 shows that the British had a dominance in carronades over the less powerful French obusiers and Spanish howitzers before the van was cut off. After, they had 3:2 numerical dominance when 26 obusiers of the van were lost. The Spanish howitzers were plunging weapons of less power than the obusiers in the mêlée. The British did not count them in a ship's rating. Adkin's description

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[14] Ibid: 225.
[15] Ibid: 392.
[16] Adkin M. The Trafalgar Companion: 227.
[17] Danielski, J. The Carronade. MilitaryHistoryNow.com. 6 Jan 2019.

of the effect of one of Victory's carronades raking the Bucentaure states "the French gunners were cut down in swathes."[18]

The extra power of the carronade will be considered in a secondary number after the direct primary number of pounds-of- metal-per-ship. This, however, introduces a problem, since it would be disingenuous to add more power for British carronades without adding something to the French and Spanish fleets for their obusiers and howitzers. The exact adjustments are, of course, debatable. Nevertheless, if they are made explicit, these can be modified in the future. Perhaps, additional information may come to light in the future or even data from tests to get more accurate estimates. To start, one may propose an initial relative power ratio of:

Carronade : Obusier : Howitzer : Cannon :: 4 : 2 : 1.5 : 1

(i.e. a carronade has 4 x power of cannon of same lb., obusier 2 x power of cannon, etc.)

Even more aspects might be addressed. The 68-pounder carronade could be loaded with both shot and a keg of 500 musket balls, even more metal.[19] However, each of the weapons of both fleets could also be loaded with various projectiles. Thus, these aspects are variable, could be made to cancel and probably would have little effect on the power ratio.

Table 2 shows the results for the secondary number for the British and Combined fleets:

British: 62,208 lbs. Combined: 70,088 lbs. Ratio: 8:9 favors Combined fleet.

The ratio is more equal with the consideration of anti-personnel weapons in the mêlée.

Lanchester's equations

In 1916, Lanchester analyzed past battles to develop quantitative relationships between opposing forces by comparing starting numbers, effective rates of fire and outcomes.[20] He paid

[18] Adkin M. The Trafalgar Companion: 225.
[19] Ibid.: 225.

special attention to Trafalgar. His relations have become known as the Lanchester's equations. They are analytical guides used at war colleges.

Rate of attrition during battle.

The equations give the rate of attrition of one's force during a modern battle. In classical and medieval times before cannon, only soldiers at the front of the phalanx or cohort could fight in one-on-one combat. Attrition occurred only there. With long range weapons, armies began to concentrate[21] firepower on the enemy in depth. Attrition affected the entire force. This was most obvious at sea, where the effect of terrain was less.

Lanchester's attrition relations become clear through an example. At Trafalgar, attrition of the Combined Fleet was due to the number of British guns firing times their percentage hits per unit time (firing effect). The same held in reverse for attrition of British guns. Note that attrition of Combined Fleet guns reduces their ability to induce attrition in the British during the next round. These relations give difference equations, where "Δ" means "difference":

Δ(FR-SP guns) / Δ(time) = - (BRIT firing effect) * (number BRIT guns).

Δ(BRIT guns) / Δ(time) = - (FR-SP firing effect) * (number FR-SP guns).

The difference equations generate numbers for "loss" in the Excel Tables shown below. The formulas used in these worksheets are shown in APPENDIX A. These relations also can be set into the "Lanchester equations," for a precise estimate at any instantaneous time point and for a solution of overall firepower. These are the differential equations shown immediately below. Their solutions require more calculation and are derived in APPENDIX B.

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[20] Lanchester FW. Aircraft in Warfare: The Dawn of the Fourth Arm. Constable & Co. Ltd, London. 1916. Reprint Bibliolife, www.bibliolife.com/opensource. 2009: 272 pp.

[21] These are equations for fire aimed at targets, not just fire at sectors of the battlefield.

Excel Worksheets – Difference equations for sizes of opposing fleets.

Tables 3, 4 and 5 show results during each hour for different conditions: Case #1 unequal sizes, Case #2 unequal sizes & gunnery skills and Case #3 unequal gunnery skills.

Table 3 shows the simplest Case #1 of unequal sizes of opposing forces. It illustrates the difference between the classical duel at right versus concentrated aimed fire at left. In the duel (pre-Lanchester) the number of guns engaged with each other is a constant, here 200 or about 10% of the starting force. This is determined by those in contact with each other, a few ships at a time. If Trafalgar were such a battle, there would be equal losses of 200 guns on each side for each hour. The attrition would be symmetrical until the very last hour when only 54 guns remained in the Combined Fleet. At the end, the British would have 296/2150 or 14% of their

	CONCENTRATED AIMED FIRE					CLASSIC DUEL ONE-ON-ONE			
TIME(T)	BRIT	loss	FR-SP	loss	ASSUME	*BRIT*		*FR-SP*	
	Navy	-αF	Navy	-βB	BRIT destroys	*Navy*	*loss*	*Navy*	*loss*
	B-guns	10%	F-guns	10%	β*B guns, β=	B-guns		F-guns	
START*	**2150**	**-185**	**1854**	**-215**	10%	*2150*	*-200*	*1854*	*-200*
60	**1965**	**-164**	**1639**	**-196**	FR-SP destroy	*1950*	*-200*	*1654*	*-200*
120	**1801**	**-144**	**1443**	**-180**	α*F guns, α=	*1750*	*-200*	*1454*	*-200*
180	**1656**	**-126**	**1262**	**-166**	10%	*1550*	*-200*	*1254*	*-200*
240	**1530**	**-110**	**1097**	**-153**	i.e. in 1st hr	*1350*	*-200*	*1054*	*-200*
300	**1421**	**-94**	**944**	**-142**	B destroys	*1150*	*-200*	*854*	*-200*
360	**1326**	**-80**	**802**	**-133**	βB of F-guns	*950*	*-200*	*654*	*-200*
	1246	**-67**	**669**	**-125**		*750*	*-200*	*454*	*-200*
	1179	**-54**	**545**	**-118**	Lanchester	*550*	*-200*	*254*	*-200*
	1125	**-43**	**427**	**-112**	attrition	*350*	*-54*	*54*	*-200*
	1082	**-31**	**314**	**-108**	equations:	*296*		*0*	
	1051	**-21**	**206**	**-105**	dB/dt=-αF				
	1030	**-10**	**101**	**-103**	dF/dt=-βB				
	1020		**0**						

Table 3. Lanchester analysis simulation of Trafalgar. Case #1. Unequal force sizes, but equal gunnery skill. At right are the results of a classical duel (pre-Lanchester) with a few ships engaging at a time on a limited front. At left are Lanchester results of concentrated aimed fire by entire navies engaging simultaneously. B-guns, F-guns are “stated guns” for ships.

guns remaining, the Combined zero. This 14% is near the 16% advantage they started with.

At the l[illegible] of Table 3, Lanchester [illegible]gs are different with concentrated aimed fire of entire navies. In the 1st hour, the 16% difference in numbers produces about 16% difference in attrition losses, 215 versus 185. But after 3 hours the attrition losses no longer differ by only 16% b[illegible] 32%, 166 ve[illegible]r 6 hours there is a 66% difference, 133 versus 80. The small starting difference in losses increases dramatically over time. There is a greater attrition in the smaller force due to more guns in the bigger force. Also, as the smaller force rapidly gets even smaller, it produces less and less attrition in the bigger force. The smaller force is caught in a losing spiral. At the end of the battle, there are 1020 remaining of the original 2150 British guns or 47%. This is a much higher survival rate compared with 14% in the duel.

Lanchester sowed that concentrated aimed fire with modern long-range weaponry changes the dynamics of battle. A small difference in initial numbe[illegible] into victory but also into survival of half one's force to fight another day. What happens when we

TIME(T)	BRIT Navy B-guns	loss -αF	FR-SP Navy F-guns	loss -βB	ASSUME Each Fleet destroys α or β percent of of their targets/hr. ie., in 1st hr. B destroys βB of F-guns
START*	2150	-93	1854	-323	
60	2057	-77	1532	-309	
120	1981	-61	1223	-297	
180	1920	-46	926	-288	
240	1873	-32	638	-281	
300	1841	-18	357	-276	
360	1824	-4	81	-274	
	1820		0		α=0.05 β=0.15**

Table 4. Lanchester analysis. Case #2. Unequal force sizes & unequal gunnery skill. Actual Trafalgar Conditions Simulation after cutting off the Van with 3:1 gunnery advantage of British. * minutes; ** 3:1 gunnery advantage of British : French Spanish.

Table 4 shows Lanchester analysis of the actual conditions from the start when Nelson cuts the line and isolated the Van. At the start, Nelson reduced the Combined Fleet from 2624 cannon to 1854 cannon. Right from the beginning, Nelson stacked the odds in his favor. When the French-Spanish Combined Fleet fire on the British, the attrition produced by the FR-SP guns in the BRIT Fleet is assumed to be 1/20 or 0.05 times their 1854 F-guns for the first hour. That is, the F-guns target and successfully eliminate 93 B-guns. The British lose 93 guns in the first hour. The French-Spanish Fleet, however, loses much more. Since the British gunnery is three times faster, the attrition produced by the British B-guns in the Combined Fleet is 3/20 or 0.15 times the 2150 B-guns for the first hour. The British concentrated fire eliminates 323 guns of the Combined Fleet with their faster firing rate x more guns. Although most of this effect is due to British gunnery skill, there are also more British guns firing.

This markedly *asymmetrical* rate of attrition continues over the subsequent hours in this simulation until the guns of the Combined fleet are abolished. Due to the larger attrition of enemy guns, those opposing the British become fewer and fewer. As a result, losses of the British become less and less, so at the end the British guns are mostly preserved intact, 1820 or 85%.

	CONCENTRATED AIMED FIRE			
TIME(T)	BRIT	loss	FR-SP	loss
	Navy	-αF	Navy	-βB
	B-guns	5%	F-guns	15%
START*	**2150**	**-108**	**2150**	**-323**
60	**2043**	**-91**	**1828**	**-306**
120	**1951**	**-76**	**1521**	**-293**
180	**1875**	**-61**	**1228**	**-281**
240	**1814**	**-47**	**947**	**-272**
300	**1766**	**-34**	**675**	**-265**
360	**1733**	**-21**	**410**	**-260**
	1712	**-8**	**150**	**-257**
	1705		**0**	

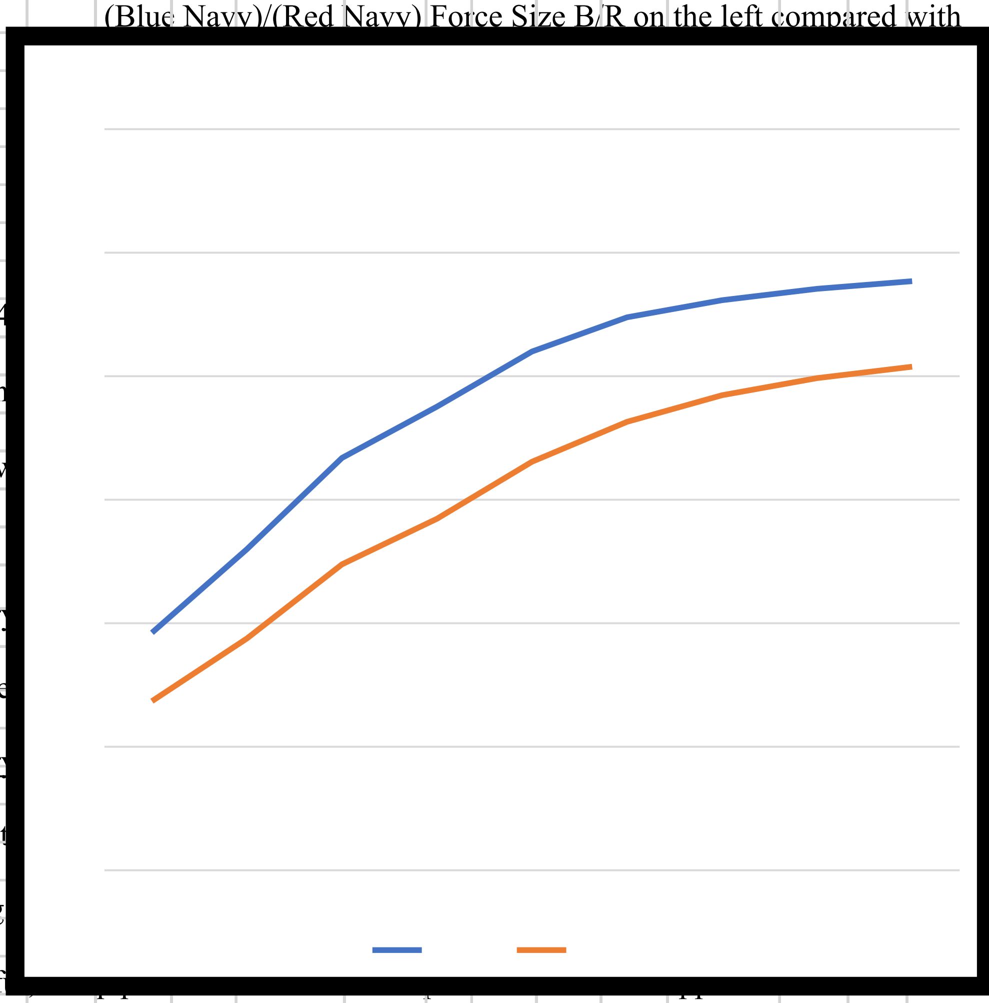
Table 5. Lanchester analysis. Case #3. British 3:1 gunnery advantage with same force sizes. 79% survive.

Case #3 or unequal gunnery skill alone is considered alone in Table 5, where both fleets start with the same size of 2150 guns. Here the British have superior gunnery without a numerical advantage. For every French-Spanish salvo, the British fire three. With a 3:1 gunnery advantage, the outcome is not much different from Table 4. This is not surprising. The British 300% gunnery skill advantage would likely dominate their 16% size advantage. Nevertheless, Nelson ran the gauntlet to give his fleet additional size advantage. He cut off the van to even the numbers. However, as will become apparent later, this tactic markedly increased his firepower advantage over his foe.

Table 6 shows a direct comparison between force size (B/R) and gunnery skills (β/α). It shows survivors at the end of the battle for ratios (Δ) from 1.1:1.0 up to 4.0:1.0 for:

	B	R	Δ=	survivors		β	α	Δ=	survivors	
1	1100	1000	1.1	424	39%	0.11	0.10	1.1	273	27%
2	1200	1000	1.2	625	52%	0.12	0.10	1.2	375	38%
3	1400	1000	1.4	936	67%	0.14	0.10	1.4	496	50%
4	1600	1000	1.6	1203	75%	0.16	0.10	1.6	571	57%
5	2000	1000	2.0	1683	84%	0.20	0.10	2.0	663	66%
6	2500	1000	2.5	2241	90%	0.25	0.10	2.5	728	73%
7	3000	1000	3.0	2776	93%	0.30	0.10	3.0	769	77%
8	3500	1000	3.5	3304	94%	0.35	0.10	3.5	797	80%
9	4000	1000	4.0	3819	95%	0.40	0.10	4.0	816	82%

(Blue Navy)/(Red Navy) Force Size B/R on the left compared with



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together in one massed number—do not divide one's force.

Table 6. Lanchester analyses. Differences in Force Size (upper left) compared with Differences in Gunnery (upper right) on Survivors. B=Blue Navy size, R=Red Navy size, Δ=ratio of B/R for 9 battles. β=effectiveness of Blue Navy gunnery, i.e. proportion of shots that hit a Red ship. α=effectiveness of Red Navy gunnery. Δ=β/α for 9 battles. Graphs below show size dominates with a plateau effect at a ratio of 4.0

The Solution to Lanchester's Equations.

The underlying relationships of Lanchester's equations have clear, direct causes stated above. The workings of the difference equations are easily shown in the Excel Worksheets. The differential equations also have been solved and one solution is given in APPENDIX B, largely derived from the Internet. The final result is well known and consistent with the Excel worksheets presented.

The Lanchester solution is the military effectiveness of a force. The solution for the B-guns is βB^2 and for the F-guns is αF^2. These are objective indicators of firepower. Just as we expect from Table 6, force size tends to dominate the picture. This is Lanchester's N-Square Law. Size dominates due to the B^2 term in βB^2. If βB^2 is larger than αF^2, then the firepower of the BRITISH is larger than that of the FRENCH and SPANISH, and the BRITISH fleet is predicted to win the battle.

Trafalgar – the effect of isolating the van through Lanchester's lens.

How did cutting off the van affect Trafalgar? Lanchester's N-Square Law shows this clearly. Before Nelson cut off the van, the force sizes were 33:27 or about 1.2:1. But the British gunnery advantage was so large at 3:1 that it dominated. This is obvious from Table 6. In the upper left of the Table, a size ratio of 1.2 preserves 52% of the fleet. But in the upper right, a gunnery ratio of 3.0 preserves 77%. British gunnery dominated the picture. Nelson cut off the van to prevent Villeneuve from having any size advantage to offset British gunnery. What additional effects did it have?

The van had 10 ships. Table 7 shows the number of ships, number of Guns+, the primary number (metal lb 1º) and secondary number (metal lb 2º) for the total pounds-of-metal of the

BRIT and of the FR-SP fleets. The British had primary lbs. 49,578 and secondary 62,208. When Nelson cut off the Van, the FR-SP decreased 30% both primary 66,164 lbs. to 45,560 (by 31%) and secondary 70,088 lbs. to 49,548 (29%). The size advantage shifted to Nelson, who already

	BRITISH				FRENCH-SPANISH				
	ships	guns+	metal lb 1º	metal lb 2º		ships	guns+	metal lb 1º	metal lb 2º
				Before Nelson cut off the Van					
B	27	2,225	49,578	62,208	F	33	2,759	66,164	70,088
F/B						1.22	1.24	1.33	1.13
βB^2	3.13	3.13	3.13	3.13	αF^2	1.49	1.54	1.78	1.27
Ratio $\beta B^2/\alpha F^2$	**2.10**	**2.04**	**1.76**	**2.46**					
				After Nelson cut off the Van					
B	27	2,225	49,578	62,208	F	23	1,945	45,560	49,548
B/F	1.17	1.16	1.09	1.25					
βB^2	4.31	4.20	3.71	4.93	αF^2	1.00	1.00	1.00	1.00
Ratio $\beta B^2/\alpha F^2$	**4.31**	**4.20**	**3.71**	**4.93**					
Ratio ↑factor	**2.06**	**2.06**	**2.11**	**2.00**					

Table 7. Lanchester Equation Solutions: Firepower before & after isolating the van.
β=3.13, α=1.00. In calculating βB^2 and αF^2, B and F are first "normalized," e.g. after the cut, B metal lb. 2º is divided by F, so 62,208/49,548=1.25. βB^2 uses B metal lb. 2º = 1.25 and αF^2 uses F metal lb. 2º = 1.00.
"metal lb. 1º "refers to pounds-of-metal/fleet primary number, "metal lb 2º " to secondary number

had the gunnery edge. Table 7 shows that by isolating 30% of the enemy, Nelson doubled his firepower ratio on the remaining opposition (Ratio ↑factor, from 2.46 to 4.93). This factor of two appears for ships, guns+ and pounds of metal. The surprising increase in power arises from the square term in the solution to the Lanchester equations. This firepower ratio of 4:1 would have allowed Nelson to conserve 95% of his fleet to protect England, if the Van did not return.

Did Nelson know that his tactic would actually double his firepower to a 4:1 ratio in the mêlée? The Lanchester equations were a century in the future, so probably not. But he knew that his tactic would increase his edge and that every edge would help. He seized every advantage he could.

Could Nelson have gained an even greater advantage by cutting off the 16 forward ships, cutting the line exactly in half? That would have increased his firepower ratio further. On the other hand, it would have delayed his raking of the Bucentaure and Villeneuve, leaving intact longer the enemy's command and control. Nelson decided to attack the command and control at the start of the Battle.

Did Nelson have to stand with all his shining medals on the quarterdeck, a perfect target for a sniper? Did Caesar need to send away his horse before battles were joined? These were great personal risks, but both acts electrified their forces and galvanized their resolve. Caesar escaped for a decade the risks of his leadership style. Sadly, fate took only minutes to catch up with Nelson.

The Navy was vital to Britain, an island nation. Nelson certainly saw his duty to ensure British supremacy on the sea and preserve the Navy intact. We can also trust that he had forged a sacred bond with his officers and men, to ensure the maximum number of survivors to man the Navy for their return home. After all, in the heat of combat, it is allegiance to one's comrade that motivates a soldier on land or sea to lay down his life for another. And even as Lord Admiral, this Nelson did.

A Century Later

Trafalgar was a textbook. Four twentieth century battles show how later Commanders studied it and applied its lessons: Tsushima, Cape Esperance, Surigao Strait and Samar.

Tsushima, Japan 1905

During the Russo-Japanese War, this battle was fought between an Imperial Russian Naval Force and the Imperial Japanese Navy (IJN). The stakes were no less than China and the Pacific.

Background.

Russia and Japan sought dominance over the riches of Chinese ports. Russia needed a year-round harbor clear of ice, Port Arthur. Japan fought to maintain her sphere of influence.

Battle. 27-28 May 1905

Figure 9. Battle of Tsushima

Admiral Togo led the IJN of 48 ships against Admiral Rozhestvensky's Imperial Russian Force of 28, arriving from its long journey across the Atlantic and Indian Ocean through the East China Sea. Both fleets had steel battleships with rifled cannon. Due to the uncertainty of friendly ports for refueling on the longer route east of Japan, Rozhestvensky took the shortest route to Vladivostok through the Tsushima Strait, between Japan and Korea (Figure 9). The IJN was waiting for the tired Russian crews. Togo repeatedly crossed Rozhestvensky's T. With these maneuvers and superior numbers, Togo annihilated the Russian Force, sinking 21 enemy ships.

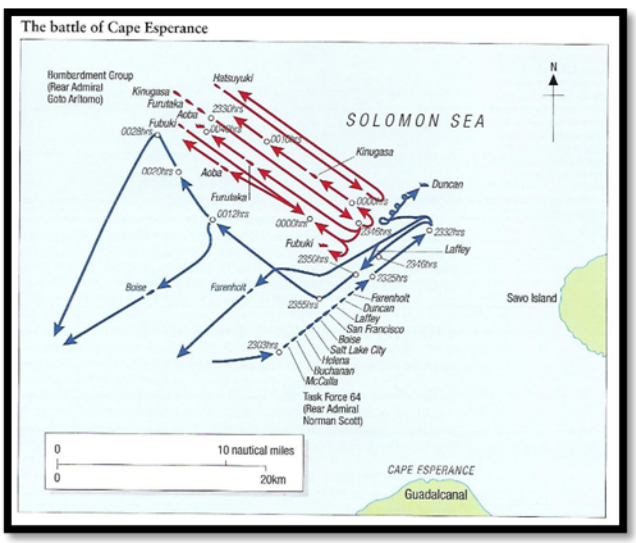
Cape Esperance, Guadalcanal, Solomon Islands, South Pacific 1942

This was the first night naval battle the US Navy won in the Second World War. It protected the strategic Henderson Field Marine Air Base from a destructive bombardment. It used the tactic of Crossing the T as a force multiplier. A smaller force repulsed a larger force.

Background.

Japan fought the Allies over control of the strategic South Pacific Solomon Islands, the route to Australia and access to oil-rich Indochina and Indonesia. The Marine airbase at Henderson Field on Guadalcanal was essential to control the airspace of the Solomons. The Imperial Japanese Navy (IJN) sent a bombardment group under Admiral Goto to knock out Henderson Field's war planes to protect an invasion force landing to occupy Guadalcanal.

Battle. 11-12 October 1942



Admiral Scott led US Navy Task Force 64 of 9 ships (blue) against Goto's larger Bombardment Group of 11 ships (red) in Figure 10. Scott crossed Goto's T, sinking two with a loss of one of his own. He protected the area and saved Henderson Field, Guadalcanal and the South Pacific. Despite inferior numbers, crossing the T repulsed a larger force. This engagement shows a dramatic effect of Crossing the T, where an inferior force prevented a superior force from achieving their objective.

Figure 11. Battle of Cape Esperance

Figure 10. Battle of Cape Esperance.

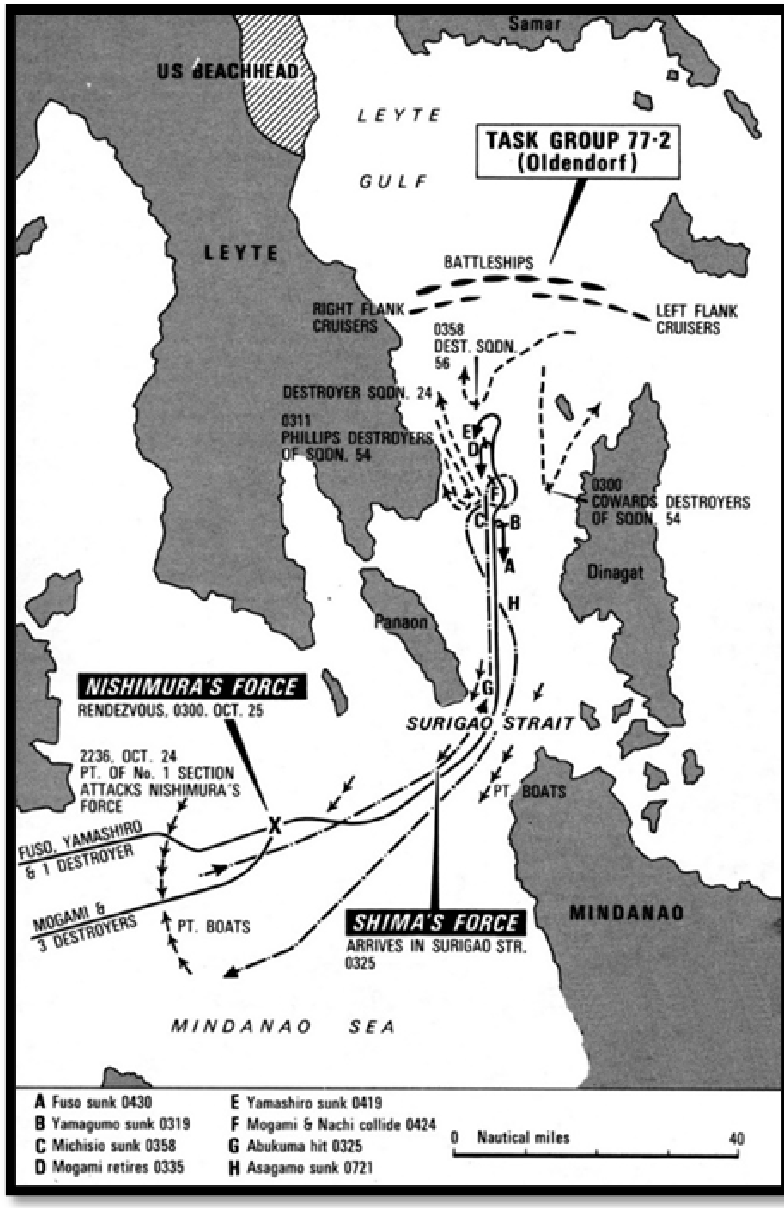
Surigao Strait, Leyte, Philippines 1944

This was the last battle where battleship fought battleship in a classic Crossing the T.

Background.

The IJ Navy fought with determination to repulse the US Navy and maintain control of the Philippines, during the last phases of the Second World War.

Figure 11.
Battle of Surigao Strait



Battle 24-25 October 1944.

Nishimura and Shima's forces approached Surigao Strait from the South (Figure 11). Only Nishimura's 7 ships proceeded north. On the way up, R. Adm. Oldendorf first ordered an attack by PT boat torpedoes, mostly ineffective. Next, destroyer torpedoes took out most of Nishimura's force. When Oldendorf detected 3 ships in range, he crossed their T, destroyed a battleship, but only harmed a cruiser and scared away a destroyer. The success of destroyer torpedoes heralded the end of the battle line.

As at Trafalgar, opposing ships are compared by pounds of metal thrown in one minute. Reasonable estimates can be obtained from Wikipedia and compared with standard sources.[22] [23] [24]

[25] Table 8 shows how estimates are made for four ships types at Surigao and Samar. These calculation methods are used in Table 9 for the ships at the Battle. Table 9 (upper right) shows Yamashiro, Mogami and Shigure survived the Destroyer attack, with 83,365 lbs. of metal/min to throw.

Armament	size	lbs.	number	rate/min	lbs./min
Pennsylvania BB (Battleship)					
Rifled gun	14-inch	1402	12	1.5	25,236
Rifled gun	5-inch	52.59	16	15	12,622
Bofors AA	40 mm	2	40	120	9,600
Oerlikon autocannon	20 mm	0.25	51	285	3,634
					51,091
Louisville CA (Heavy Cruiser)					
Rifled gun	8-inch	335	9	4	12,060
Rifled gun	5-inch	52.59	8	15	6,311
Bofors	40 mmg	2.00	28	120	6,720
Oerlikon	20 mm	0.25	54	285	3,848
					28,938
Columbia CL (Light Cruiser)					
Rifled gun	6-inch	130	12	9.0	14,040
Rifled gun	5-inch	52.59	12	15	9,466
Bofors	40 mmg	2.00	16	120	3,840
Oerlikon	20 mm	0.25	13	285	926
					28,272
Fletcher Class DD (Destroyer)					
Rifled gun	5-inch	52.59	5	15	3,944
Bofors	40 mm	2.00	8	120	1,920
Oerlikon	20 mm	0.25	9	285	641
Torpedo Mark 15	21-inch	825	10		8,250
					14,756

U.S.N.				
OLDENDORF Left Flank TF 79			Class	
Louisville CA			Northampton	
Portland CA			Portland	
Minneapolis CA			New Orleans	
Denver CL			Cleveland	
Columbia CL			Cleveland	
Newcomb DD		Smoot	Fletcher	5.10.7
Leary DD		Smoot	Fletcher	5.10.7
Grant DD		Smoot	Fletcher	5.10.7
Robinson DD			Fletcher	5.4.4.
Halford DD			Fletcher	5.6.11
Bryant DD			Fletcher	5.4.4.
Edwards DD			Gleaves	
Bennion DD			Fletcher	5.4.4.
Leutze DD			Fletcher	5.10.7
WEYLER Battle Line TF 79				
West Virginia BB			Colorado	
Maryland BB			Colorado	
California BB			Tennesse	
Tennessee BB			Tennesse	
Mississippi BB			New Mexico	
Pennsylvania BB			Pennsylvania	
Claxton DD			Fletcher	5.10.7
Cony DD			Fletcher	5.10.7
Thorn DD			Gleaves	4.4.5.
Aulick DD			Fletcher	5.4.4.
Sigourney DD			Fletcher	5.4.4.
Welles DD			Gleaves	4.4.7.
BERKEY Right Flank TF 79				
Shropshire CA, HMAS			County	
Phoenix CL			Brooklyn	
Boise CL			Brooklyn	
Hutchins DD		McManes	Fletcher	5.10.7
Daly DD		McManes	Fletcher	5.10.7
Bache DD		McManes	Fletcher	5.10.7
Arunta DD, HMAS		McManes	Tribal	
Killen DD		McManes	Fletcher	5.4.4.
Beale DD		McManes	Fletcher	5.10.7
Remey DD		Cow(1)	Fletcher	5.10.7
McGowan DD		Cow(1)	Fletcher	5.10.7
Melvin DD		Cow(1)	Fletcher	5.10.7
Mertz DD		Cow(2)	Fletcher	5.10.7
McDermut DD		Cow(1)	Fletcher	5.10.7
Monssen DD		Cow(1)	Fletcher	5.10.7
McNair DD		Cow(2)	Fletcher	5.10.7
PT BOATS	39	-1	2062	
TOTAL POUNDS OF METAL/MIN				
			SHORT TON	
			LONG TON	
			METRIC TONNE	
BBCACL	14			
DD	28	42		
PT	39	81		

[22] Morison SE. Op.cit.

[23] Hornfischer JD. The Last Stand of the Tin Can Sailors. Bantam Books, Random House, New York, 2004: 500 pp.

[24] Stille M. USN Destroyer vs IJN Destroyer. The Pacific 1943. Osprey Publishing, London. 2012: 80 pp.

[25] www.bosamar.com

Table 9 is set up like the tables for Trafalgar. It adds up the total pounds of metal per minute that each opposing force could throw, and potentially land on target. The U.S. Navy ships are listed on the left, the Imperial Japanese ships on the right. For the U.S.N. the name of each ship is given with its Class. For specific destroyer divisions their commanders are given, such as Capt. Smoot. The armament of some Fletcher-Class Destroyers differed from others, revealed by the column before pounds of metal. A “5.10.7.10” indicates five 5-inch rifled guns, ten Bofors 40 mm anti-aircraft guns, seven Oerlikon 20 mm autocannon and ten Mark 15 torpedoes.

The I.J.N. had different types of armament, including 5.5-inch guns and the Type 93 Long Lance Torpedo. As with the U.S.N., pounds of metal/minute thrown was added up for each individual ship.

There is one number for pounds of metal for the U.S.N., 978,527 lbs. There are three numbers for the I.J.N. The first is Nishimura’s and Shima’s forces combined, totaling 260,488 lbs. The second is when Nishimura proceeded alone, with a force totaling 168,213 lbs. (upper right). The third is after the destroyer attack but before Oldendorf crossed his Nishimura’s T, when three ships totaled a nominal 83,365 lbs. (upper right). The last likely overestimates their throwing metal, since Yamashiro was substantially damaged, Mogami less so.

No adjustments in firepower are made on either side for the added explosive charges of shells or torpedoes. The variables to be considered are armor piercing versus high explosive shells, etc. This is feasible with more detailed accounting. More variables would be added. This may skew the balance somewhat but would not be likely to change the conclusions.

U. S. NAVY (U)						IMPERIAL JAPANESE NAVY (J)			
	BBCACL	BBCACL + DD	BBCACL + DDPT	metal lbs. /min			BBCACL	BBCACL + DD	metal lbs. /min
Before Shima turns back									
U	14	42	81	978,527		J	6	14	260,488
U/J	2.33	3		3.76					
υU^2	5.44	9		14.1		γJ^2	1.00	1.00	1.00
Ratio $\upsilon U^2/\gamma J^2$	**5.44**	**9.00**		**14.11**					
After Shima turns back									
U	14	42	81	978,527		J	3	7	168,213
U/J	4.67	6.00		5.82					
υU^2	21.78	36.00		33.84		γJ^2	1.00	1.00	1.00
Ratio $\upsilon U^2/\gamma J^2$	**21.78**	**36.00**		**33.84**					
Ratio ↑	**4.00**	**4.00**		**2.40**					
After Destroyer Attack									
U	14	42	81	978,527		J	2	4	83,365
U/J	7.00	10.50		11.74					
υU^2	49.00	110.25		137.78		γJ^2	1.00	1.00	1.00
Ratio $\upsilon U^2/\gamma J^2$	**49.00**	**110.25**		**137.78**					
Ratio ↑ ASAD	**2.25**	**3.06**		**4.07**					
Ratio ↑ BSAD	**9.00**	**12.25**		**9.76**					

Table 10. Firepower υU^2 (U.S.N.) and γJ^2 (I.J.N.) at key times Battle of Surigao Strait

Table 10 shows overwhelming U.S.N. firepower compared to I.J.N. firepower; 1) before Shima turns back, 2)after Shima turns back and 3) after the Destroyer attack. This leaves little question who will win the Battle. At the start, there is a 3:1 ship ratio and a 14:1 firepower ratio. It is unlikely that any stratagem would have overcome such a dominant force.

When Yamashiro BB, Mogami CA and Shigure DD came in range, the first two had been damaged by the destroyer forces. The massed firepower on Yamashiro is shown in Figure 12.

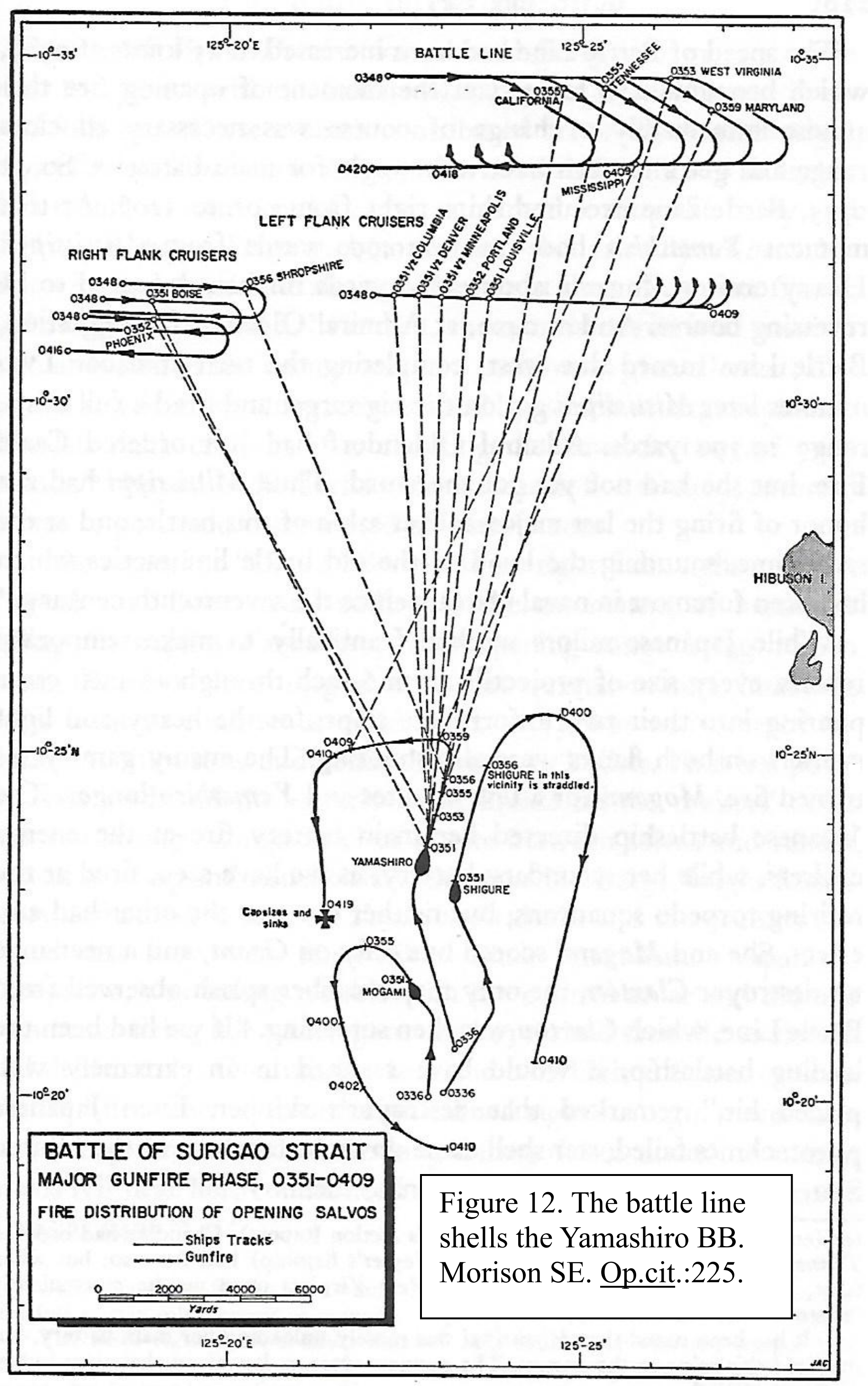


Figure 12. The battle line shells the Yamashiro BB. Morison SE. Op.cit.:225.

The Crossing of the T was a coup-de-grâce on the already ailing Yamashiro. Oldendorf used the battle line and cruisers on those ships, which got by the destroyers' torpedoes, according to current naval tactical doctrine.[26] Note that the "terrain" of the Surigao Strait was a force multiplier. It kept Shima back and thus divided the I.J.N. force. It was also essentially a "defile". Nishimura's force was decimated by fire "en defilade" by the destroyers and PT boats. Then the Strait funnel the remaining ship into the fire "en enfilade" by the battle line Crossing the T (Figure 1). Yamashiro was sunk. Mogami and Shigure escaped the T. Repeated air strikes eventually destroyed Mogami. Shigure got away from the Surigao Straits. Although this was a classic Crossing the T maneuver in a virtually set piece battle by overwhelming odds, it really illustrated the effectiveness of the destroyer torpedoes.

The narrative now shifts to the next and last instance of Crossing the T reviewed here. From the story of a massive battle line shelling three ships in the Surigao Straits one goes east off Samar Island, 4 hours later, where a few tin cans attack Goliath.

[26] The classic use of destroyers was to launch an offensive torpedo attack before heavy ships were within gunfire range. This was the doctrine of the Naval War College in Newport, R.I. Morison SE. History of United States Naval Operations in World War II. Vol 12. Leyte. June 1944-January 1945. Naval Institute Press, Annapolis, Maryland. 1953, 2011:213.

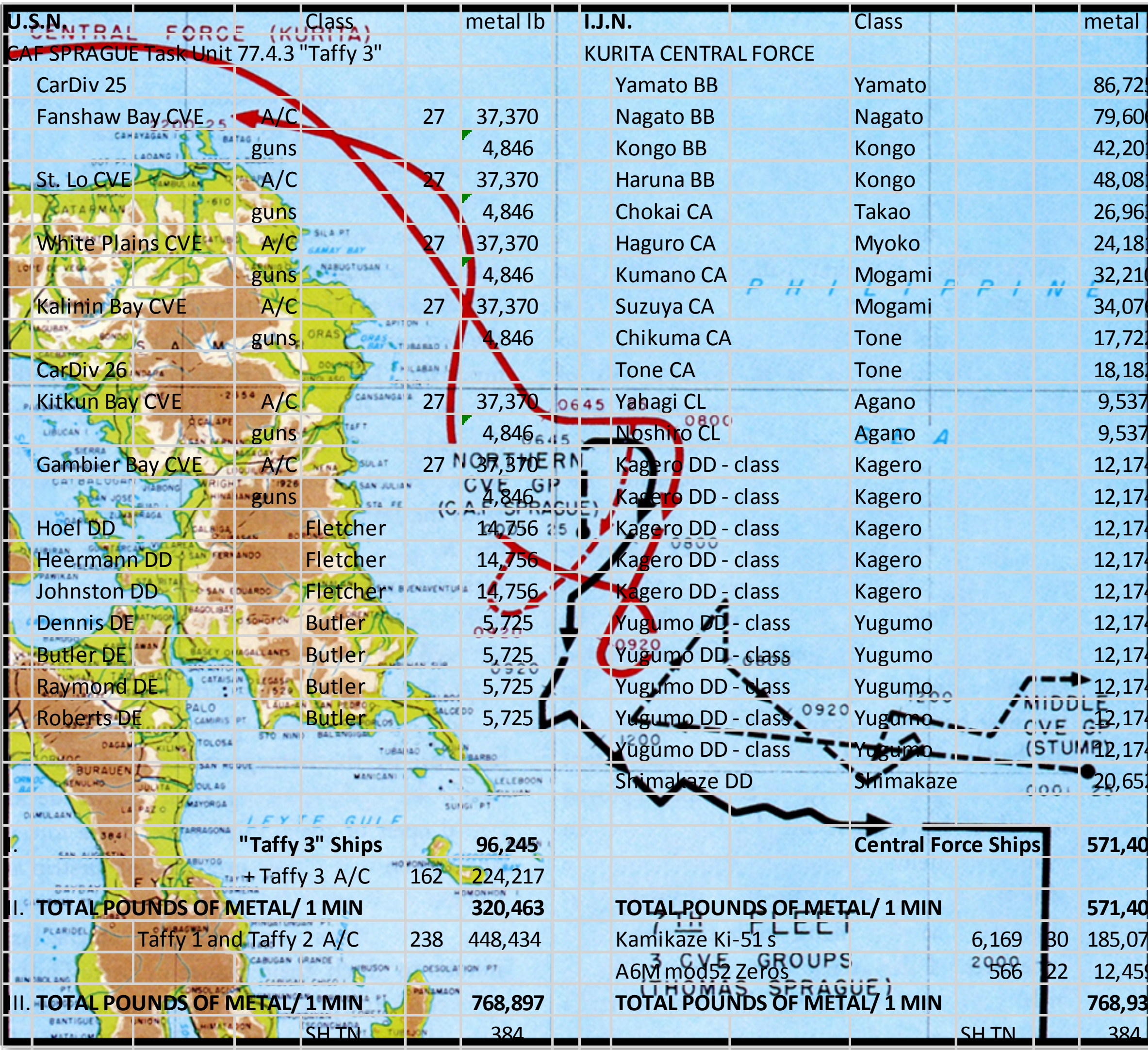
Battle off Samar, Leyte, Philippines 1944

An action during this battle illustrates one destroyer crossing the T of a destroyer squadron, to repel them from the carriers. One ship against four or seven…

Background.

The I.J.N. sent Vice Admiral Kurita with a Battleship Force east through the San Bernardino Strait between the southern tip of Luzon and the northern tip of Samar Island (Figure 13). His mission was to ambush and destroy sixteen (16) US Navy Escort Carriers and 400 aircraft east of Samar Island. Admiral Halsey had taken the entire Third Fleet north to chase a Decoy Fleet, without leaving any picket to monitor the strait. Kurita surprised the Escort Carriers and their defensive force Task Group 77.4, comprised of Task Units "Taffy 1, Taffy 2 and Taffy 3." The initial action occurred between Admiral Kurita's Central Force of 4 battleships, 6 heavy cruisers, 2 light cruisers and 11 destroyers versus R. Adm. C.A.F. Sprague's "Taffy 3" of 6 escort carriers with 162 aircraft, protected by only 3 destroyers and 4 destroyer escorts (Table 11).

Out of courage and necessity, all members of Taffy 3 fought strenuously for survival. Commander Evans' action on the destroyer Johnston is recounted in detail due to its successful use both of torpedoes against a heavy cruiser and of Crossing the T to repel a destroyer squadron. This allows comparison with the action 4 hours before and 70 miles west in the Surigao Strait.



U.S.N.	Class		metal lb	I.J.N.	Class			metal lb
CAF SPRAGUE Task Unit 77.4.3 "Taffy 3"				KURITA CENTRAL FORCE				
CarDiv 25				Yamato BB	Yamato			86,72
Fanshaw Bay CVE	A/C	27	37,370	Nagato BB	Nagato			79,60
	guns		4,846	Kongo BB	Kongo			42,20
St. Lo CVE	A/C	27	37,370	Haruna BB	Kongo			48,08
	guns		4,846	Chokai CA	Takao			26,96
White Plains CVE	A/C	27	37,370	Haguro CA	Myoko			24,18
	guns		4,846	Kumano CA	Mogami			32,21
Kalinin Bay CVE	A/C	27	37,370	Suzuya CA	Mogami			34,07
	guns		4,846	Chikuma CA	Tone			17,72
CarDiv 26				Tone CA	Tone			18,18
Kitkun Bay CVE	A/C	27	37,370	Yahagi CL	Agano			9,537
	guns		4,846	Noshiro CL	Agano			9,537
Gambier Bay CVE	A/C	27	37,370	Kagero DD - class	Kagero			12,17
	guns		4,846	Kagero DD - class	Kagero			12,17
Hoel DD	Fletcher		14,756	Kagero DD - class	Kagero			12,17
Heermann DD	Fletcher		14,756	Kagero DD - class	Kagero			12,17
Johnston DD	Fletcher		14,756	Kagero DD - class	Kagero			12,17
Dennis DE	Butler		5,725	Yugumo DD - class	Yugumo			12,17
Butler DE	Butler		5,725	Yugumo DD - class	Yugumo			12,17
Raymond DE	Butler		5,725	Yugumo DD - class	Yugumo			12,17
Roberts DE	Butler		5,725	Yugumo DD - class	Yugumo			12,17
				Yugumo DD - class	Yugumo			12,17
				Shimakaze DD	Shimakaze			20,65
I.	"Taffy 3" Ships		96,245		Central Force Ships			571,409
	+ Taffy 3 A/C	162	224,217					
II. TOTAL POUNDS OF METAL/ 1 MIN			320,463	TOTAL POUNDS OF METAL/ 1 MIN				571,409
Taffy 1 and Taffy 2 A/C		238	448,434	Kamikaze Ki-51 s		6,169	30	185,070
				A6M mod52 Zeros		566	22	12,459
III. TOTAL POUNDS OF METAL/ 1 MIN			768,897	TOTAL POUNDS OF METAL/ 1 MIN				768,938
	SH TN		384			SH TN		384
	LG TN		343			LG TN		343
	MT TN		349			MT TN		349
CVE	6			BB&CA	12			
DD	3			DD	11			
DE	4							
A/C	400			A/C	30			

Figure 13. Battle off Samar. CAF Sprague Northern CVE GP is Taffy 3. Map shows general location and track of forces. Source www.wikipedia.org. Detailed locations for specific ships shown in Hornfischer JD. Op.cit. San Bernardino Strait upper left off map

Table 11 shows the ships of the opposing forces, analogous to Table 9 for the Surigao Strait. For

Table 11. Battle off Samar - Ships. U.S.N. United States Navy. I.J.N. Imperial Japanese Navy. CVE Escort Carrier. DD Destroyer. DE Destroyer Escort. U.S.N. A/C Wildcats F4F Fighters and Avenger TBM Torpedo Bombers. Metal lbs. amount of ordnance in pounds thrown at enemy by each type. Total pounds of metal/ 1 min = maximum pounds ordnance on enemy in time period (min), under ideal conditions. I.J.N. A/C Ki-51 slow single engine piloted bomb. SH TN short ton. BB battleship. CA heavy cruiser. CL light cruiser. Total pounds of Metal for Each Phase of Battle: I. Initial Attack. II. Taffy 3 A/C airborne. III. Taffy 1 & 2, Kamikaze + other Japanese A/C airborne. Morison SE. Op.cit.: 242-316. www.wikipedia.org.

Table 11 shows the pounds of metal for the three phases of the battle: I. Destroyers against the Central Force, II. + Taffy 3's A/C airborne and finally III. + Taffy 1+2, Kamikaze and other I.J.N. A/C.

Battle 25 October 1944.

Commander Ernest E. Evans,[27] in the spirit of Nelson, steered the destroyer Johnston at flank speed to fire on the cruiser squadron flagship Kumano, hitting its superstructure and blowing off its bow with ten Mark 15 torpedoes, two of which hit. The Japanese commander was dumbstruck, believing he was under attack by a larger force. The Johnston received much damage. The Fletcher-class destroyer had little armor, leading to its nickname "tin can." Despite this, it kept fighting and also hit the battleship Haruna and cruiser Haguro.[28] This audacity at least partly explains the otherwise long initial phase, despite the I.J.N. overwhelming odds then.

Then Evans ordered a daring Crossing the T maneuver. Figure 14 shows the ship positions near the time of the action. Evans saw five Japanese ships closing to attack the escort carriers. The light cruiser Yahagi was followed in line by the four destroyers Urakaze, Isokaze, Ukikaze and Nowaki, in Desron 10. Evans fired 12 hits on Yahagi, which then veered to the right and broke off action. Evans shifted fire to the first destroyer and began to cross its T. All the destroyers in column, apparently fearing this classic maneuver, released their torpedoes prematurely, which failed to hit, and turned 90 degrees to the right,[29] [30] away from the carriers. Other sources state that Evans crossed the T of seven destroyers in two columns,[31] probably

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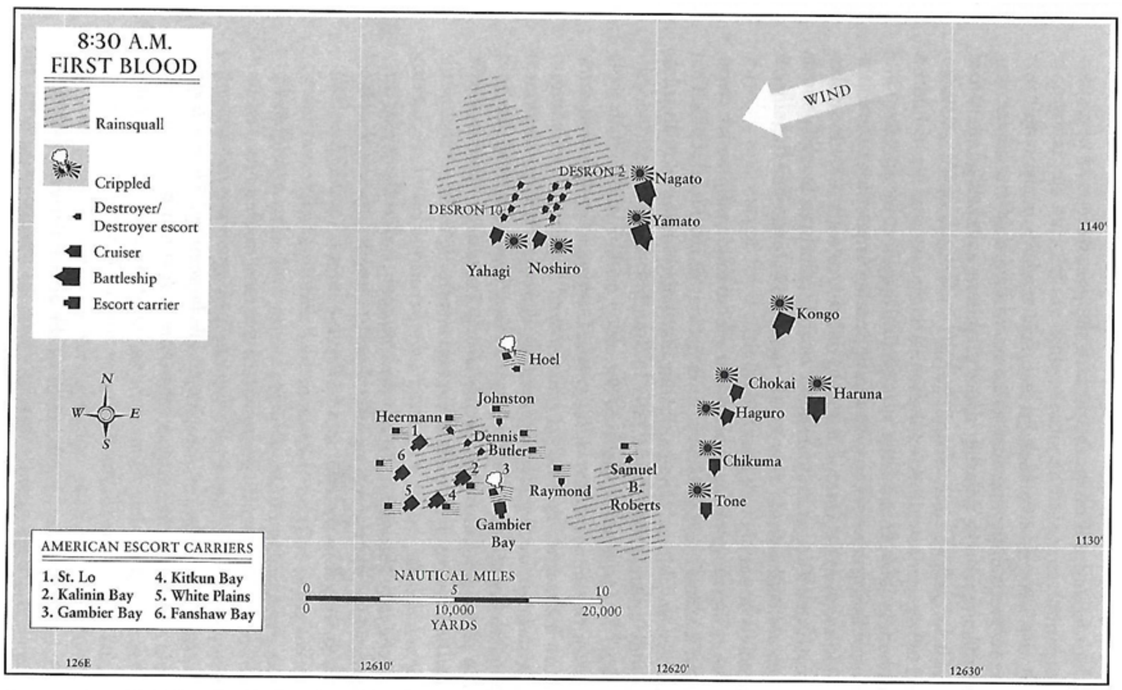
[27] Native American Ancestry: Half Cherokee, Quarter Creek. Graduated U.S. Naval Academy 1931, where Trafalgar and Tsushima in syllabus. www.wikipedia.org
[28] Morison SE. History of United States Naval Operations in World War II. Vol 12. Leyte. June 1944-January 1945. Naval Institute Press, Annapolis, Maryland. 1953, 2011:255-258
[29] Morison SE. History of United States Naval Operations in World War II. Vol 12. Leyte. June 1944-January 1945. Naval Institute Press, Annapolis, Maryland. 1953, 2011: 271-272.
[30] Hornfischer JD. Op.cit.: 274-275

Noshiro's Desron 2 (Figure 14). The many vigorous actions by Johnston that day eventually cost her and her skipper's life but shielded the carriers.



As planes from Taffy 2 and Taffy 3 strafe and bomb Kurita, the end nears for the *Hoel,* dead in the water as the Japanese close in. The escort carrier *Gambier Bay* is hit too, loses steam, and drops out of formation. Sprague orders his destroyer screen to intercept cruisers looming on his port quarter. The *Haruna,* ranging out to the southeast, opens fire on Taffy 2.

Figure 14. Battle off Samar. Ship positions as Johnston fires on Yahagi, before crossing T of destroyer squadron. Johnston just left of center, Yahagi and Noshiro above. CVE Escort Carriers to left of Johnston. Hornfischer JD. Op.cit.: 279.

[31] https://en.wikipedia.org/wiki/Battle_off_Samar#USS_Johnston.

U. S. NAVY (U)						IMPERIAL JAPANESE NAVY (J)			
	DDDECVEg	A/C^3	A/C^{123}	metal lbs. /min			BBCACLDD	Ki-51s A6Ms	metal lbs. /min
Central Force Ships attack Taffy 3									
U	9			**96,245**		J	23		**571,409**
U/J						J/U	2.6		5.9
υU^2	1.0			1.0		γJ^2	6.5		35
Ratio $\gamma J^2/\upsilon U^2$							**6.5**		**35**
Taffy 3 A/C airborne									
U	9	162		**320,463**		J	23		**571,409**
U/J						J/U	2.6		1.8
υU^2	1.0			1.0		γJ^2	6.5		3.2
Ratio $\gamma J^2/\upsilon U^2$							**6.5**		**3.2**
Ratio ↓ by factor of									-11
Taffy 1 and Taffy 2 A/C airborne, Ki-51 Kamikaze and other I.J.N. A/C strikes									
U	9	162	400	**768,897**		J	23	52	**768,938**
U/J			7.7			J/U	2.6		1.0
υU^2	1.0		59	1.0		γJ^2	6.5	1.0	1.0
Ratio $\gamma J^2/\upsilon U^2$			**59**				**6.5**		1.0
[redacted]									

Table 12. Battle of Samar. Firepower at each phase: I. Initial attack, II. Taffy 3 A/C airborne and III. Taffy 1 & 2, Kamikaze and other I.J.N. A/C airborne.

There was a large imbalance of force at the start of the battle. Table 11 shows the U.S.N. destroyers were dwarfed by I.J.N. ships of the line and more destroyers. U.S.N. ships were outnumbered 23 to 9. The small armament on the 6 CVE's are counted most optimistically as equivalent to 2 destroyers.[32] Table 12 shows that in total pounds of metal the Japanese had a 5.9 to 1 advantage at the start. Thus, they had a 35-fold firepower advantage due to the N-square law.

[32] The CVE guns were effective and did account for hits on the Japanese ships.

During this Initial Phase the Destroyers were shielding the Escort Carriers, which were trying to get their planes in the air. It was the most critical phase. The aggressive actions of the Destroyers stunned and confused the Japanese command and control. During the delay Taffy 3 launched her Aircraft. The planes tripled the pounds of metal per min thrown of Taffy 3. This reduced the firepower advantage of the Japanese by a factor of eleven (x11). When Taffy 1 and Taffy 2 got their planes up, the odds evened out even though Japanese aircraft arrived. Even when the planes ran out of ammunition, their strafing runs spooked the Central Force, which eventually retired.

Despite the imbalance in the forces for the first two phases of the battle, the I.J. Navy force was repulsed. The Japanese lost 3 heavy cruisers and 52 aircraft. The US lost 2 escort carriers, 2 destroyers, 1 destroyer escort and 23 aircraft. Overall, the force multiplier of audacity during an unexpected counterattack by small vulnerable ships[33] against large armored ships caused enough confusion and delay in the I.J.N. command and control to allow the carriers to launch their aircraft.

Johnston's crossing the T of a destroyer squadron deflected 4-7 ships from the carriers. This local action shows a 1:4 or 1:7 force multiplier, greater than the 9:11 effect at Cape Esperance, where a 9-ship battle line deflected 11 ships. Johnston's action shows the largest effect of crossing the T to the author's knowledge.

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[33] Fletcher-class Destroyer has ¼ inch steel hull thickness, which can be penetrated by a low caliber projectile.

Conclusion

Trafalgar's lessons were studied by naval tacticians during the century of the Pax Britannica. Its lessons led to Crossing the T and the development of the Lanchester Equations. Although much of what Nelson used at Trafalgar was discovered before, he put it all together in a masterstroke. His heroic death on the bridge of the Victory and the love tragedy of Emma lived on in romance. It was his memory and the memory of the naval battle that stopped Napoleon, which imprinted itself on the minds of a gracious nation. When rifled guns, fire control, radar, torpedoes, submarines and aircraft changed naval warfare, tactics continued to evolve … but still from the Tree of Trafalgar.

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www.bosamar.com

www.google.com

www.wikipedia.org

APPENDIX A: Excel Worksheet with Formulas

EXCEL Worksheet with Formulas for Lanchester calculations in a battle of the same force sizes (2000) with differences in gunnery. B-guns have 3:1 superior gunnery over F-guns. The variables to enter for force sizes are C57 and E57. The variables for gunnery effectiveness (rate of fire and accuracy) are G57 and G60. The vertical length of the table can expand to see smaller differences or finer time intervals. All the worksheets in this paper were constructed in a similar way. TIME(T) in minutes. C70 and E70 are survivors at end of battle. E70 formula (not shown) resulted in the zero (0) shown here.

A	B	C	D	E	F	G
53		CONCENTF				
54	TIME(T)	BRIT	loss	FR-SP	loss	ASSUME
55		Navy	-αF	Navy	-βB	BRIT destroys
56		B-guns	=G60	F-guns	=G57	β*B guns, β=
57	START*	**2000**	**=-E57*D56**	**2000**	**=-C57*F56**	**0.15**
58	60	**=C57+D57**	**=-E58*D56**	**=E57+F57**	**=-C58*F56**	FR-SP destroy
59	120	**=C58+D58**	**=-E59*D56**	**=E58+F58**	**=-C59*F56**	α*F guns, α=
60	180	**=C59+D59**	**=-E60*D56**	**=E59+F59**	**=-C60*F56**	**0.05**
61	240	**=C60+D60**	**=-E61*D56**	**=E60+F60**	**=-C61*F56**	i.e. in 1st hr
62	300	**=C61+D61**	**=-E62*D56**	**=E61+F61**	**=-C62*F56**	B destroys
63	360	**=C62+D62**	**=-E63*D56**	**=E62+F62**	**=-C63*F56**	βB of F-guns
64		**=C63+D63**	**=-E64*D56**	**=E63+F63**	**=-C64*F56**	
65		**=C64+D64**	**=-E65*D56**	**=E64+F64**	**=-C65*F56**	Lanchester
66		**=C65+D65**	**=-E66*D56**	**=E65+F65**	**=-C66*F56**	attrition
67		**=C66+D66**	**=-E67*D56**	**=E66+F66**	**=-C67*F56**	equations:
68		**=C67+D67**	**=-E68*D56**	**=E67+F67**	**=-C68*F56**	dB/dt=-αF
69		**=C68+D68**	**=-E69*D56**	**=E68+F68**	**=-C69*F56**	dF/dt=-βB
70		**=C69+D69**		**0**		

APPENDIX B: Solving Lanchester's Equations.

The Lanchester equations for concentrated aimed fire are:

To solve these equations and get the results for firepower βB^2 and αF^2, it's necessary to understand the derivative, represented by and , and then the 'integral', which just reverses the operation of the derivative. Those acquainted with this know it as calculus.

APPENDIX B consists of two parts. First, a brief review of the concepts of derivative and integral, copied from the clearest pedagogical sources that the author could find.[34] [35] [36] Second, a clear derivation of the solution to the Lanchester equations, from a book review of Lanchester's work.[37]

If you are familiar with derivative and integral, go to "Lanchester Equation Solutions."

Usually a general introduction to the derivative and integral takes a semester in University. However, there are examples of focused applications, for instance, early in courses in Physics, where a well-written and illustrated chapter reviews the essentials, even as students are beginning their calculus course.[38]

The text here is not original but was extracted from the internet and standard textbooks. Once understood, the concept will get us to the results for firepower.

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[34] en.wikipedia.org, www.google.com,

[35] Feynman RP, Leighton RB, Sands M. The Feynman Lectures on Physics. Vol I: Mainly Mechanics, Radiation and Heat. Addison-Wesley. Reading, MA. 1963: 8.5-8.10

[36] Thomas GB. Calculus and Analytic Geometry. Addison-Wesley. Reading, MA. 3Ed. 1966:26-39 155-164.

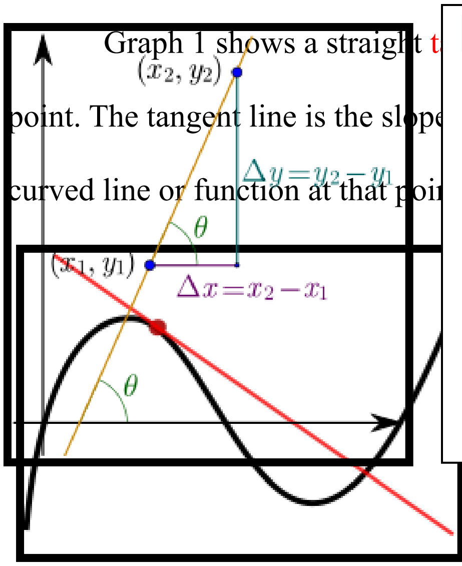
[37] www.amazon.com/books Lanchester FW. Aircraft in Warfare. Comment Viktor Blasjo FN below.

[38] Feynman RP et al. Op.cit.

Derivative.

This is a brief application of just enough of the derivative and integral, borrowing generously from Wikipedia and standard sources, to get a solution of Lanchester's equations. There are many clear introductions, including Thomas and the first chapters of Feynman (see References). The derivative of a straight line is its slope.

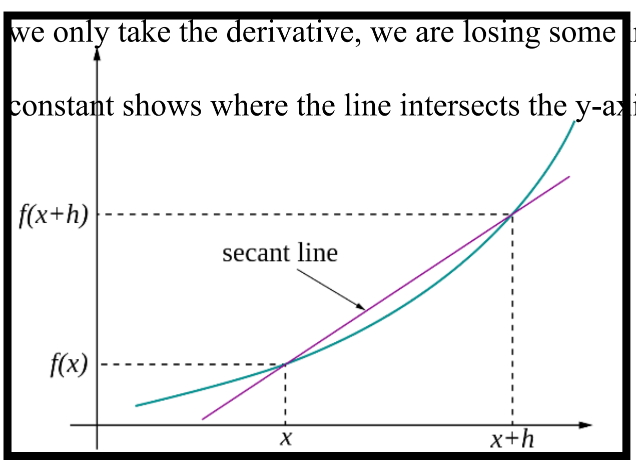
Graph 1 shows a straight t ... **ved line (black)** at one point. The tangent line is the slope ... lso the derivative of the curved line or function at that poi...



Graph 2. Slope of a linear function:

... drawn in ... nction, ... gent line ... nction at ... on ... below.

Graph 2 shows the details of the slope of a **line**, or linear function. The slope is the change in y divided by the change in x, at the point x. This would be $\Delta y = y_2 - y_1$ divided by $\Delta x = x_2 - x_1$. The slope is $m = \Delta y / \Delta x$. The slope of the line is its derivative. But you can see that if we only take the derivative, we are losing some information about the line, the constant *b*. This constant shows where the line intersects the y-axis below or above the origin.

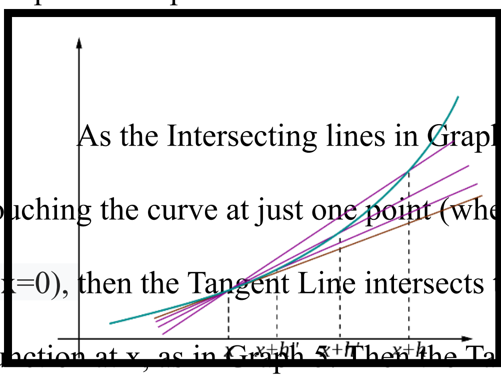


Graph 3. The line intersects a parabola $y=f(x)=x^2$ at (x) and (x+h).

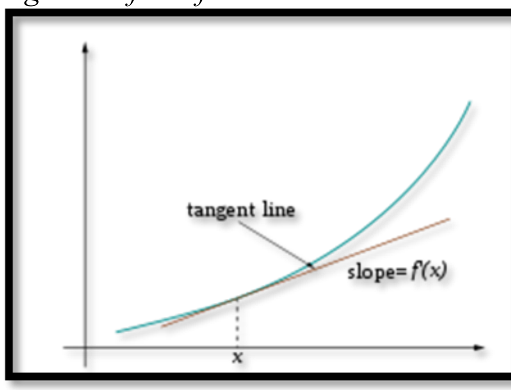
In the measurement of the slope of the line there is a visible distance between x_1 and x_2. So, $\Delta x = x_2 - x_1$ is more than just one point, it is a line segment on the x axis. In the figure it is the base of a right triangle.

We can only use this type of approach on a straight line, where the slope is the same at all points along the line. Using a right triangle won't work on a curved line, where the slope changes from one part of the curve to the next.

Graph 3 shows a **line** intersecting a **parabola** at the points x and x+h. We can estimate the slope or derivative as (f(x+h) – f(x)) / ((x+h) – (x)), but this is obviously not the derivative for any *one* point on the curved line of the parabola. Finding the slope of the tangent line at any *one* point on a parabola is where *we really need a general form for the derivative*.



Graph 4. The intersecting line gets closer to the tangent line at (x) as:
(x+h) becomes a smaller or
(x+h) approaches (x) or
(h) approaches (0) or
(Δx→0).
When Δx=0, the tangent line intersects one point on $y = x^2$, the parabola.



Graph 5. Tangent intersects f(x) when Δx=0. Derivative is f(x).

As the Intersecting lines in Graph 4 near touching the curve at just one point (when Δx=0), then the Tangent Line intersects the function at x, as in Graph 5. Then the Tangent Line becomes the slope of the curve and is its derivative at that point.

Derivative as a Limit.

The derivative for $y=x^2$ is the value of the slope of the tangent line at a specified point, at the point x on the parabola above in Graph 5. The numerical value changes or is variable depending on the value of x on the parabola. The derivative is the limit of the variable when the line goes from Graph 4 to Graph 5. The equation to find the value of the derivative is:

Just as the graphs above showed the lines getting closer and closer to a tangent line intersecting one point on the curve, where (x+h) became smaller and smaller until it approached (x) and ($h \to 0$), the algebraic quantity is made to do the same thing. The quantity is evaluated as (. The Limit of is calculated as (. The slope of the tangent line goes from the simple situation of intersecting *multiple points* to the precise situation of intersecting just *one point* on a parabola. At that point, $x_2 = x_1$,so, $\Delta x = x_2 - x_1 = 0$, or . By a series of successive approximations, the slope is calculated as $\Delta x = x_2 - x_1$ goes from multiple points to just one point where The equations below show the steps leading to the answer:

Now (x2) = (x1+Δx) or (x+ Δx) and (x1) = (x), so the above equation becomes:

Multiplying it all out, one gets:

Canceling out the terms:

Dividing by :

As , the goes to zero, and the Limit becomes **2x**. This is the final answer for the derivative of $y = x^2$. This is the instantaneous or point derivative, the slope of a tangent line intersecting only one point of the function **$y= x^2$**.

Note that the derivative can be obtained from most any variable x. It can be distance (*dx*) or time (*dt*). In the Lanchester equations the variable x is time (*dt*) and the variable y is what is changing with time (*dB*), number of ships, guns or pounds-of-metal.

Integral.

The Integral is obtained by reversing the operation for the derivative. Lanchester solves his equation in a certain way by infinitesimals. The more common way today is by integration.

In reversing the operation, remember that obtaining the derivative took away a constant *b*. When calculating the integral, one must put back a placeholder for that constant, C. The symbol for the integral[39] is a long, sinuous S, . The Integral sums up the values of a function over a range of x's. The integral acting on a function f(x) looks like:

The little *dx* at the end indicates that the Integral is summing up f(x) over many small changes in x (or *dx's*), a range of x's. If one then lets the derivative be the function inside the Integral, it shows a clear picture of what's really going on.

The same derivative analyzed above can be put inside the new Integral symbol just introduced here. When that is done, this is what it looks like:

Then the (*dx*) on the right end of the Integral and the (dx) in the denominator of the derivative will be able to cancel out to give a simple expression:

The Integral and derivative symbols are now side by side on the right. They operate in opposite ways. They cancel each other out to give the original function f(x) = y , but one must also add back the placeholder constant:

[39] The process to calculate the derivative is called differentiation, to calculate the integral integration.

In summary, the Integral of the derivative gives the original function $y = x^2$, with a placeholder C as needed.

That is outline of the Derivative and Integral as far as it is applied in the Lanchester equations.

Solutions to Lanchester's Equations.

The derivative and integral can solve the Lanchester equations:

Victor Blasjo has clarified steps in a solution, in a review of Lanchester on the internet.[40]

These steps are repeated here. If you understand the footnote, no need to read further…

Otherwise:

Dividing the first by the second equation, the *dt* and minus sign drop out:

Rearranging:

Using the Integral to sum up and get rid of the *dB* and *dF* terms (β and α are constants):

By integrating:[41]

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[40] Viktor Blasjo. www.amazon.com/books comment: Lanchester FW. Aircraft in Warfare. 1 Jan 2007: "Personally, I was interested only in a particular part of this book, namely Lanchester's battle model. It goes like this. There is a battle between two armies, one with A soldiers and one with B soldiers. Each army has a constant efficiency coefficient (determined by weaponry, training, etc.): a side A soldier takes out a enemies per unit time while a side B soldier takes out b enemies per unit time. The battle is then described by the differential equations dA/dt = - bB and dB/dt = - aA. Dividing the first by the second gives aAda = bBdb, which we integrate to get $aA^2 - bB^2$ = constant. (Lanchester avoids mentioning integration and uses a direct infinitesimal argument.) The sign of this constant determines the outcome of the battle, since if, for example, there are side A troops still standing when B reaches zero then the constant must be positive (indeed we see that the number of side A troops surviving the battle can be calculated by setting B=0 and solving for A). Also, the fact that the strength of an army is proportional to the square of its size has an important strategical implication: never divide your forces. For example, assuming equal efficiency a=b=1, an army of 5000 could handle an army of 7000 split into two, $5000^2=4000^2+3000^2$, but if the 5000 army faced the full 7000 army at once it would be destroyed after having killed only about 2100 enemies, $7000^2 - 5000^2 = 4900^2$."

[41] When one does the integral, the results are actually (1/2)βB2 and (1/2)αF2. But one can easily get rid of the factor (1/2) by adjusting the constant without changing the fundamental solutions or relationships.

Subtracting from both sides:

These are the solutions to the Lanchester differential equations. The terms βB^2 and αF^2 are the military effectiveness or firepower of the B-guns and F-guns, respectively. If βB^2 is larger than αF^2, then the military effectiveness of the B Navy is larger than the F Navy and the constant C is positive, and B should win the battle. The term βB^2 shows that military effectiveness is proportional to β in a linear fashion, but proportional to the square of B. This is the Lanchester N-Square Law. In other words, the difference in force sizes will usually dominate the outcome. We suspected this from the Excel worksheets in specific cases, but here it is proven in general. This has been recognized for a long time. “Do not divide one’s force in battle.” “Divide and conquer.” “Defeat in detail.”